Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

Go Online! Take the Chapter Readiness Quiz online as another option

Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Graph and label each point in the coordinate plane.

1. $W(5, 2)$

2. $X(0, 6)$

3. $Y(-3, -1)$

4. $Z(4, -2)$

5. **GAMES** Carolina is using the diagram to record her chess moves. She moves her knight 2 spaces up and 1 space to the left from f3. What is the location of the knight after Carolina completes her turn?

Find each sum or difference.

6. $\frac{2}{3} + \frac{5}{6}$

7. $2\frac{1}{18} + 4\frac{3}{4}$

8. $\frac{13}{18} - \frac{5}{9}$

9. $14\frac{3}{5} - 9\frac{7}{15}$

10. **FOOD** Alvin ate $\frac{1}{3}$ of a pizza for dinner and took $\frac{1}{6}$ of it for lunch the next day. How much of the pizza does he have left?

Evaluate each expression.

11. $(-4 - 5)^2$

12. $(6 - 10)^2$

13. $(8 - 5)^2 + [9 - (-3)]^2$

Solve each equation.

14. $6x + 5 + 2x - 11 = 90$

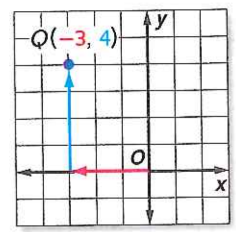
15. $8x - 7 = 53 - 2x$

QuickReview

Example 1

Graph and label the point $Q(-3, 4)$ in the coordinate plane.

Start at the origin. Since the x-coordinate is negative, move 3 units to the left. Then move 4 units up since the y-coordinate is positive. Draw a dot and label it Q.



Example 2

Find $3\frac{1}{6} + 2\frac{3}{4}$.

$3\frac{1}{6} + 2\frac{3}{4} = \frac{19}{6} + \frac{11}{4}$ Write as improper fractions.

$= \frac{19}{6}\left(\frac{2}{2}\right) + \frac{11}{4}\left(\frac{3}{3}\right)$ The LCD is 12.

$= \frac{38}{12} + \frac{33}{12}$ Multiply.

$= \frac{71}{12}$ or $5\frac{11}{12}$ Simplify.

Example 3

Evaluate the expression $[-2 - (-7)]^2 + (1 - 8)^2$.

Follow the order of operations.

$[-2 - (-7)]^2 + (1 - 8)^2$

$= 5^2 + (-7)^2$ Subtract.

$= 25 + 49$ $5^2 = 25, (-7)^2 = 49$

$= 74$ Add.

Get Started on the Chapter

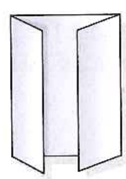
Go Online! for Vocabulary Review Games and key vocabulary in 13 languages

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 1. To get ready, identify important terms and organize your resources. Working with a partner can be helpful as you prepare and as you read the chapter.

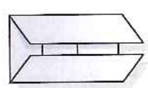
FOLDABLES® StudyOrganizer

Tools of Geometry Make this Foldable to help you organize your Chapter 1 notes about points, lines, and planes; angles and angle relationships; and formulas and notes for distance, midpoint, perimeter, area, and volume. Begin with a sheet of 11″ × 17″ paper.

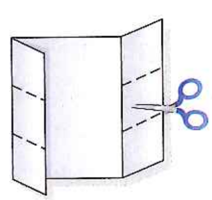
1 **Fold** the short sides to meet in the middle.



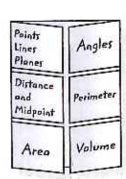
2 **Fold** the booklet in thirds lengthwise.



3 **Open and cut** the booklet in thirds lengthwise.



4 **Label** the tabs as shown.



NewVocabulary

English		Español
collinear	p. 5	colineal
coplanar	p. 5	coplanar
congruent	p. 15	congruente
midpoint	p. 26	punto medio
segment bisector	p. 27	bisectriz de segmento
angle	p. 36	angulo
vertex	p. 36	vertice
angle bisector	p. 39	bisectriz de un angulo
perpendicular	p. 48	perpendicular
polygon	p. 56	poligono
perimeter	p. 58	perimetro
volume	p. 69	volumen

ReviewVocabulary

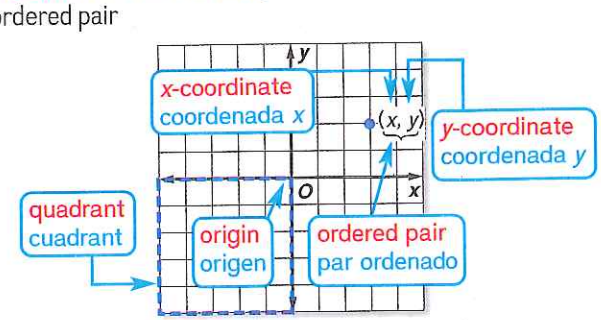
ordered pair par ordenado a set of numbers or coordinates used to locate any point on a coordinate plane, written in the form (x, y)

origin origen the point where the two axes intersect at their zero points

quadrants cuadrantes the four regions into which the x-axis and y-axis separate the coordinate plane

x-coordinate coordenada x the first number in an ordered pair

y-coordinate coordenada y the second number in an ordered pair



LESSON

1-1 Points, Lines, and Planes

Then

- You used basic geometric concepts and properties to solve problems.

Now

1. Identify and model points, lines, and planes.
2. Identify intersecting lines and planes.

Why?

- On a subway map, the locations of stops are represented by *points*. The route the train can take is modeled by a series of connected paths that look like *lines*. The flat surface of the map on which these points and lines lie is representative of a *plane*.

Targeted TEKS

G.4(A) Distinguish between undefined terms, definitions, postulates, conjectures, and theorems.

Mathematical Processes

G.1(A) Apply mathematics to problems arising in everyday life, society, and the workplace.

G.1(G) Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

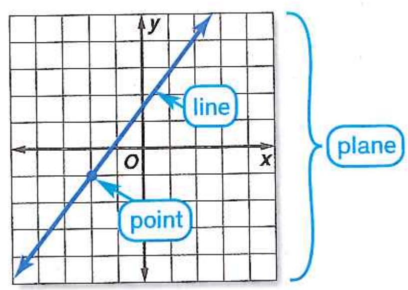
NewVocabulary

- undefined term
- point
- line
- plane
- collinear
- coplanar
- intersection
- definition
- defined term
- space

1 Points, Lines, and Planes

Unlike the real-world objects that they model, points, lines, and planes do not have any actual size. In geometry, *point*, *line*, and *plane* are considered **undefined terms** because they are only explained using examples and descriptions.

You are already familiar with the terms point, line, and plane from algebra. You graphed on a coordinate *plane* and found ordered pairs that represented *points* on *lines*. In geometry, these terms have a similar meaning.



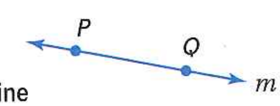
The phrase *exactly one* in a statement such as, "There is exactly one line through any two points," means that there is *one and only one*.

KeyConcept Undefined Terms

A **point** is a location. It has neither shape nor size.

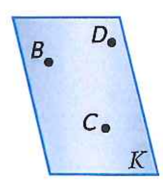
Named by	a capital letter
Example	point A

A **line** is made up of points and has no thickness or width. There is exactly one line through any two points.



Named by	the letters representing two points on the line or a lowercase script letter
Example	line m, line PQ or $\overleftrightarrow{PQ}$, line QP or $\overleftrightarrow{QP}$

A **plane** is a flat surface made up of points that extends infinitely in all directions. There is exactly one plane through any three points not on the same line.



Named by	a capital script letter or by the letters naming three points that are not all on the same line
Example	plane $\mathcal{K}$, plane BCD, plane CDB, plane DCB, plane DBC, plane CBD, plane BDC

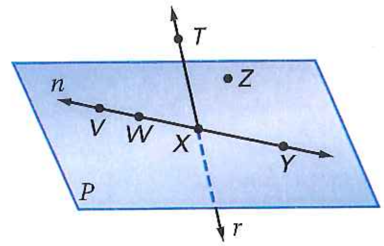
Collinear points are points that lie on the same line. *Noncollinear* points do not lie on the same line. **Coplanar** points are points that lie in the same plane. *Noncoplanar* points do not lie in the same plane.

TEKS G.4(A)

Example 1 Name Lines and Planes

Use the figure to name each of the following.

a. a line containing point *W*



The line can be named as line *n*, or any two of the four points on the line can be used to name the line.

$\overleftrightarrow{VW}$ $\overleftrightarrow{WV}$ $\overleftrightarrow{VX}$ $\overleftrightarrow{XV}$ $\overleftrightarrow{VY}$ $\overleftrightarrow{YV}$
$\overleftrightarrow{WX}$ $\overleftrightarrow{XW}$ $\overleftrightarrow{WY}$ $\overleftrightarrow{YW}$ $\overleftrightarrow{XY}$ $\overleftrightarrow{YX}$

b. a plane containing point X

One plane that can be named is plane P. You can also use the letters of any three *noncollinear* points to name this plane.

plane *XZY*	plane *VZW*	plane *VZX*
plane *VZY*	plane *WZX*	plane *WZY*

The letters of each of these names can be reordered to create other acceptable names for this plane. For example, *XZY* can also be written as *XYZ*, *ZXY*, *ZYX*, *YXZ*, and *YZX*. In all, there are 36 different three-letter names for this plane.

StudyTip

Additional Planes Although not drawn in Example 1b, there is another plane that contains point *X*. Since points *W*, *T*, and *X* are noncollinear, point *X* is also in plane *WTX*.

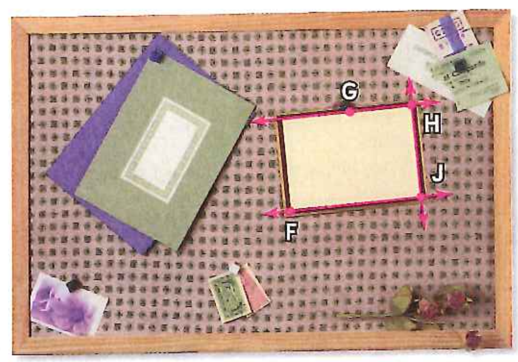
GuidedPractice

1A. a plane containing points *T* and *Z* **1B.** a line containing point *T*

TEKS G.4(A)

Real-World Example 2 Model Points, Lines, and Planes

MESSAGE BOARD **Name the geometric terms modeled by the objects in the picture.**

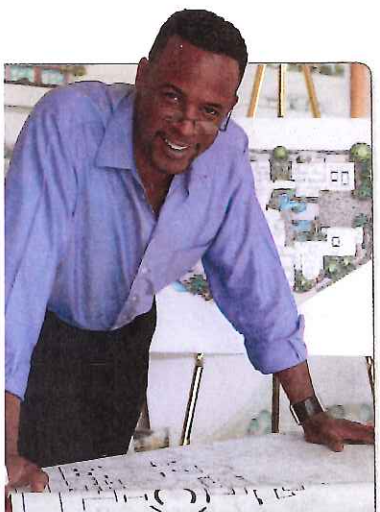


The push pin models point *G*.

The maroon border on the card models line *GH*.

The edge of the card models line *HJ*.

The card itself models plane *FGJ*.



Real-WorldCareer

Drafter Drafters use perspective to create drawings to build everything from toys to school buildings. Drafters need skills in math and computers. They get their education at trade schools, community colleges, and some 4-year colleges. Refer to Exercises 50 and 51.

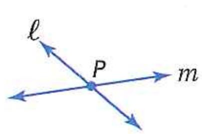
GuidedPractice

Name the geometric term modeled by each object.

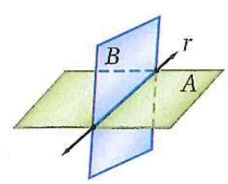
2A. stripes on a sweater **2B.** the corner of a box

2 Intersections of Lines and Planes

The **intersection** of two or more geometric figures is the set of points they have in common. Two lines intersect in a point. Lines can intersect planes, and planes can intersect each other.



P represents the intersection of lines ℓ and m.



Line r represents the intersection of planes A and B.

Example 3 Draw Geometric Figures

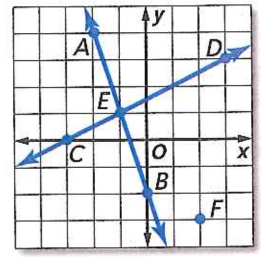
Draw and label a figure for each relationship.

a. ALGEBRA Lines AB and CD intersect at E for $A(-2, 4)$, $B(0, -2)$, $C(-3, 0)$, and $D(3, 3)$ on a coordinate plane. Point F is coplanar with these points, but not collinear with $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

Graph each point and draw $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

Label the intersection point as E.

An infinite number of points are coplanar with A, B, C, D and E but not collinear with $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. In the graph, one such point is $F(2, -3)$.

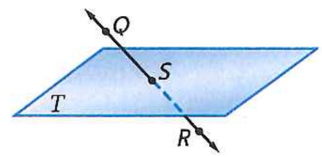


b. QR intersects plane $\mathcal{T}$ at point S.

Draw a surface to represent plane $\mathcal{T}$ and label it.

Draw a dot for point S anywhere on the plane and a dot that is not on plane $\mathcal{T}$ for point Q.

Draw a line through points Q and S. Dash the line to indicate the portion hidden by the plane. Then draw another dot on the line and label it R.



GuidedPractice

3A. Points $J(-4, 2)$, $K(3, 2)$, and L are collinear.

3B. Line p lies in plane N and contains point L.

> **StudyTip**
> **Three-Dimensional Drawings** Because it is impossible to show an entire plane in a figure, edged shapes with different shades of color are used to represent planes.

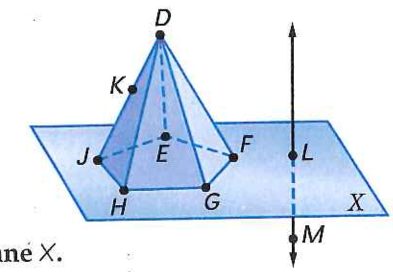
> **StudyTip**
> **Using Your Text** Notice that new terms are listed at the beginning of the lesson and also highlighted in context.

Definitions or **defined terms** are explained using undefined terms and/or other defined terms. **Space** is defined as a boundless, three-dimensional set of all points. Space can contain lines and planes.

Example 4 Interpret Drawings

a. How many planes appear in this figure?

Six: plane $\mathcal{X}$, plane JDH, plane JDE, plane EDF, plane FDG, and plane HDG.



b. Name three points that are collinear.

Points J, K, and D are collinear.

c. Name the intersection of plane HDG with plane $\mathcal{X}$.

Plane HDG intersects plane $\mathcal{X}$ in $\overleftrightarrow{HG}$.

d. At what point do $\overleftrightarrow{LM}$ and $\overleftrightarrow{EF}$ intersect? Explain.

It does not appear that these lines intersect. $\overleftrightarrow{EF}$ lies in plane $\mathcal{X}$, but only point L of $\overleftrightarrow{LM}$ lies in $\mathcal{X}$.

GuidedPractice

Explain your reasoning.

4A. Are points E, D, F, and G coplanar?

4B. At what point or in what line do planes JDH, JDE, and EDF intersect?

> **StudyTip**
> **MP Apply Math** A point has no dimension. A line exists in one dimension. However, a circle is two-dimensional, and a pyramid is three-dimensional.

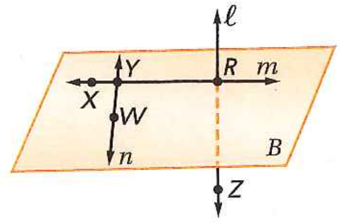
Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Example 1
TEKS G.4(A)

Use the figure to name each of the following.



1. a line containing point X

2. a line containing point Z

3. a plane containing points W and R

Example 2
TEKS G.4(A)

Name the geometric term modeled by each object.

4. a tightrope

5. a floor

Example 3

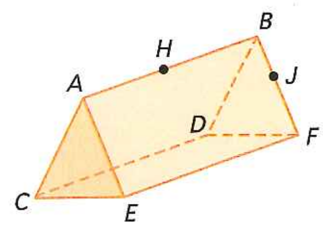
Draw and label a figure for each relationship.

6. A line in a coordinate plane contains $A(0, -5)$ and $B(3, 1)$ and a point C that is not collinear with $\overleftrightarrow{AB}$.

7. Plane Z contains lines x, y, and w. Lines x and y intersect at point V and lines x and w intersect at point P.

Example 4

Refer to the figure.



8. How many planes are shown in the figure?

9. Name three points that are collinear.

10. Are points A, H, J, and D coplanar? Explain.

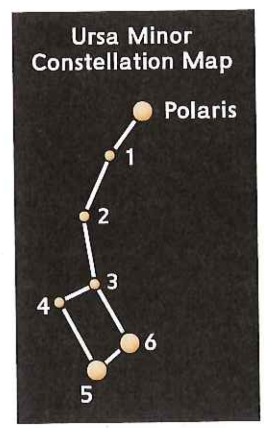
11. Are points B, D, and F coplanar? Explain.

12. ASTRONOMY Ursa Minor, or the Little Dipper, is a constellation made up of seven stars in the northern sky including the star Polaris.

a. What geometric figures are modeled by the stars?

b. Are Star 1, Star 2, and Star 3 collinear on the constellation map? Explain.

c. Are Polaris, Star 2, and Star 6 coplanar on the map?

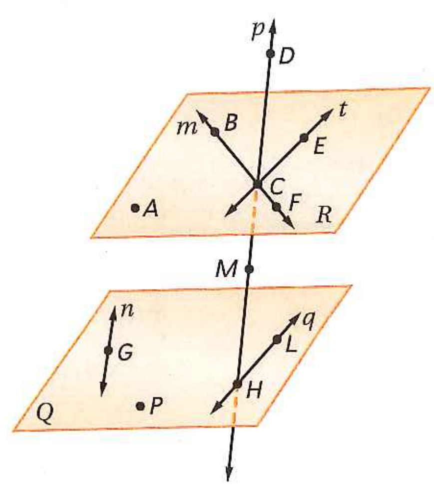


Practice and Problem Solving

Extra Practice is on page R1.

Example 1
TEKS G.4(A)

Refer to the figure.



13. Name the lines that are only in plane Q.

14. How many planes are labeled in the figure?

15. Name the plane containing the lines m and t.

16. Name the intersection of lines m and t.

17. Name a point that is not coplanar with points A, B, and C.

18. Are points F, M, G, and P coplanar? Explain.

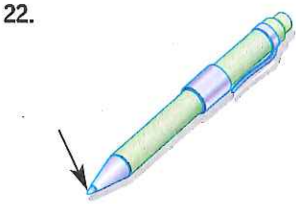
19. Name the points not contained in a line shown.

20. What is another name for line t?

21. Does line n intersect line q? Explain.

Example 2
TEKS G.4(A)

Name the geometric term(s) modeled by each object.

22. 

23. 

24. 

25. 

26. a blanket
27. a knot in a rope
28. a telephone pole
29. the edge of a desk
30. two connected walls
31. a partially opened folder

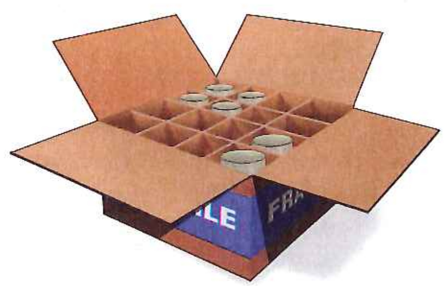
Example 3

Draw and label a figure for each relationship.

32. Line m intersects plane R at a single point.
33. Two planes do not intersect.
34. Points X and Y lie on $\overleftrightarrow{CD}$.
35. Three lines intersect at point J but do not all lie in the same plane.
36. Points $A(2, 3)$, $B(2, -3)$, C and D are collinear, but A, B, C, D, and F are not.
37. Lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{NP}$ are coplanar but do not intersect.
38. $\overleftrightarrow{FG}$ and $\overleftrightarrow{JK}$ intersect at $P(4, 3)$, where point F is at $(-2, 5)$ and point J is at $(7, 9)$.
39. Lines s and t intersect, and line v does not intersect either one.

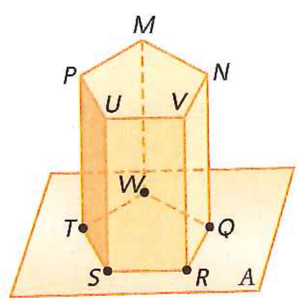
Example 4

MP APPLY MATH **When packing breakable objects such as glasses, movers frequently use boxes with inserted dividers like the one shown.**



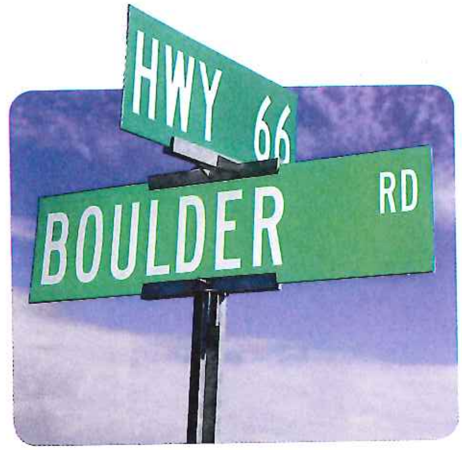
40. How many planes are modeled in the picture?
41. What parts of the box model lines?
42. What parts of the box model points?

Refer to the figure at the right.



43. Name two collinear points.
44. How many planes appear in the figure?
45. Do plane A and plane MNP intersect? Explain.
46. In what line do planes A and QRV intersect?
47. Are points T, S, R, Q, and V coplanar? Explain.
48. Are points T, S, R, Q, and W coplanar? Explain.

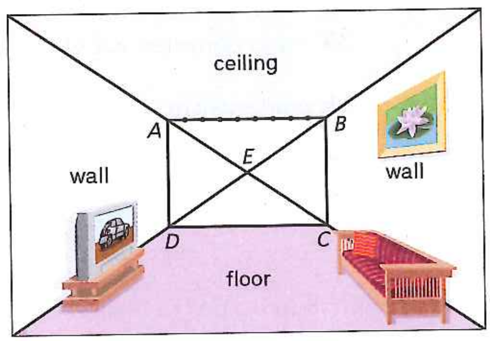
49. **FINITE PLANES** A *finite plane* is a plane that has boundaries, or does not extend indefinitely. The street signs shown are finite planes.



a. If the pole models a line, name the geometric term that describes the intersection between the signs and the pole.

b. What geometric term(s) describes the intersection between the two finite planes? Explain your answer with a diagram if necessary.

50. **ONE-POINT PERSPECTIVE** One-point perspective drawings use lines to convey depth. Lines representing horizontal lines in the real object can be extended to meet at a single point called the *vanishing point*. Suppose you want to draw a tiled ceiling in the room below with nine tiles across.

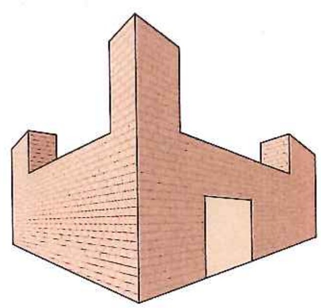


a. What point represents the vanishing point in the drawing?

b. Trace the figure. Then draw lines from the vanishing point through each of the eight points between A and B. Extend these lines to the top edge of the drawing.

c. How could you change the drawing to make the back wall of the room appear farther away?

51. **TWO-POINT PERSPECTIVE** Two-point perspective drawings use two vanishing points to convey depth.



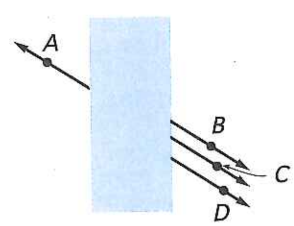
a. Trace the drawing of the castle shown. Draw five of the vertical lines used to create the drawing.

b. Draw and extend the horizontal lines to locate the vanishing points and label them.

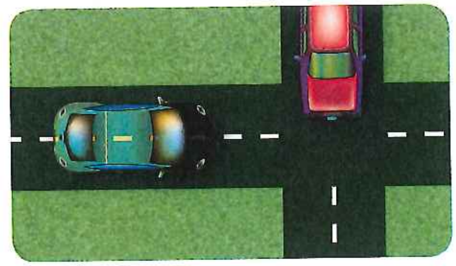
c. What do you notice about the vertical lines as they get closer to the vanishing point?

d. Draw a two-point perspective of a home or a room in a home.

52. MP **JUSTIFY ARGUMENTS** Name two points on the same line in the figure. How can you support your assertion?



53. **TRANSPORTATION** When two cars enter an intersection at the same time on opposing paths, one of the cars must adjust its speed or direction to avoid a collision. Two airplanes, however, can cross paths while traveling in different directions without colliding. Explain how this is possible.



54. **MULTIPLE REPRESENTATIONS** Another way to describe a group of points is called a locus. A **locus** is a set of points that satisfy a particular condition. In this problem, you will explore the locus of points that satisfy an equation.

   a. **Tabular** Represent the locus of points satisfying the equation $2 + x = y$ using a table of at least five values.

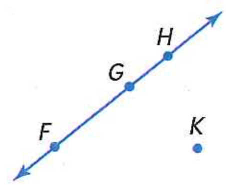
   b. **Graphical** Represent this same locus of points using a graph.

   c. **Verbal** Describe the geometric figure that the points suggest.

55. **PROBABILITY** Three of the labeled points are chosen at random.

   a. What is the probability that the points chosen are collinear?

   b. What is the probability that the points chosen are coplanar?



56. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the locus of points that satisfy an inequality.

   a. **Tabular** Represent the locus of points satisfying the inequality $y < -3x - 1$ using a table of at least ten values.

   b. **Graphical** Represent this same locus of points using a graph.

   c. **Verbal** Describe the geometric figure that the points suggest.

TEKS G.4(A)

H.O.T. Problems

Use Higher-Order Thinking Skills

57. **ORGANIZE IDEAS** Sketch three planes that intersect in a line.

58. **ERROR ANALYSIS** Camille and Hiroshi are trying to determine the most number of lines that can be drawn using any two of four random points. Is either correct? Explain.

Camille	Hiroshi
Since there are four points, $4 \cdot 3$ or 12 lines can be drawn between the points.	You can draw $3 \cdot 2 \cdot 1$ or 6 lines between the points.

59. **JUSTIFY ARGUMENTS** What is the greatest number of planes determined using any three of the points *A*, *B*, *C*, and *D* if no three points are collinear?

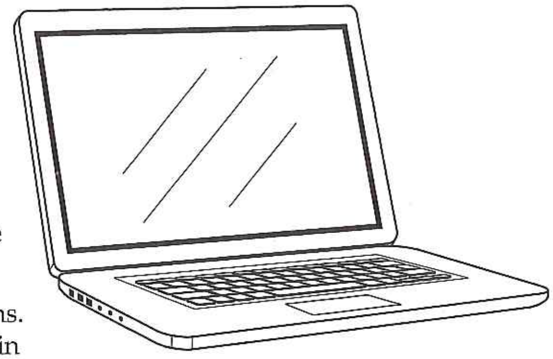
60. **JUSTIFY ARGUMENTS** Is it possible for two points on the surface of a prism to be neither collinear nor coplanar? Justify your answer.

61. **WRITING IN MATH** Refer to Exercise 49. Give a real-life example of a finite plane. Is it possible to have a real-life object that is an infinite plane? Explain your reasoning.

Preparing for Assessment

Example TEKS G.4(A) MP G.1(D), G.1(F)

TEKS REVIEW What undefined term is best modeled by the laptop screen?



A Line
B Parallel
C Plane
D Rectangle

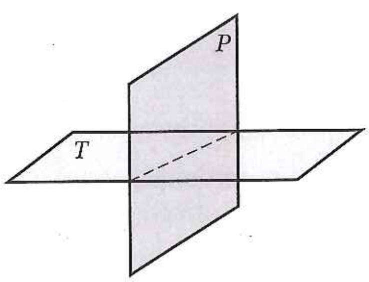
The screen is rectangular. However, the term rectangle, as well as the term parallel, are not undefined, so eliminate choices B and D.

A plane is a flat surface made up of points that extend in all directions. A line is a figure made up of an infinite number of points extending in opposite directions.

The screen best models the undefined term *plane*. The correct answer is choice C.

62. **ACT/SAT** The figure illustrates the intersection of plane P and plane T. The planes extend infinitely in all directions.

What undefined term best describes the intersection?



A Line
B Plane
C Point
D Segment
E None of these

63. **GRIDDABLE** Four lines are coplanar. What is the greatest number of intersection points that can exist? TEKS G.4(A) MP G.1(D), G.1(F)

64. Which of the following are undefined terms? TEKS G.4(A) MP G.1(D), G.1(F)

I. Distance
II. Line
III. Plane
IV. Point

F II, III, and IV
G I and III only
H II and III only
J II and IV only

65. Samir is using a compass to draw a circle on a piece of paper. He places the metal tip of the compass at one location on the paper, and then moves the pencil around the tip to draw the figure. Which of the following models a term that can be defined? TEKS G.4(A) MP G.1(D), G.1(F)

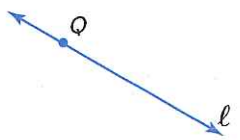
A The location of the metal tip on the paper
B The plane that contains the piece of paper
C The tip of the pencil
D The circle drawn by the pencil

EXTEND

1-1 Geometry Lab
Describing What You See

When you are learning geometric concepts, it is critical to have accurate drawings to represent the information. It is helpful to know what words and phrases can be used to describe figures. Likewise, it is important to know how to read a geometric description and be able to draw the figure it describes.

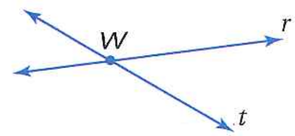
The figures and descriptions below help you visualize and write about points, lines, and planes.



Point Q is on ℓ.

Line ℓ contains Q.

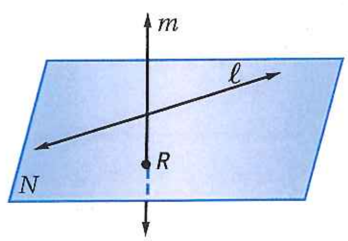
Line ℓ passes through Q.



Lines r and t intersect at W.

Point W is the intersection of r and t.

Point W is on r. Point W is on t.



Line ℓ and point R are in N.

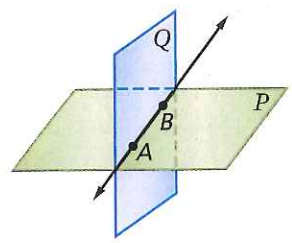
Point R lies in N.

Plane N contains R and ℓ.

Line m intersects N at R.

Point R is the intersection of m with N.

Lines ℓ and m do not intersect.



$\overleftrightarrow{AB}$ is in P and Q.

Points A and B lie in both P and Q.

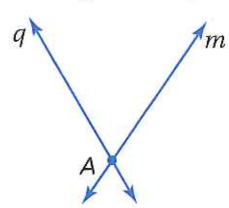
Planes P and Q both contain $\overleftrightarrow{AB}$.

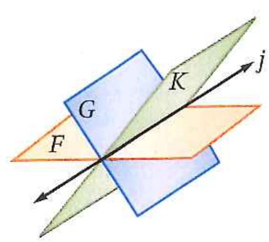
Planes P and Q intersect in $\overleftrightarrow{AB}$.

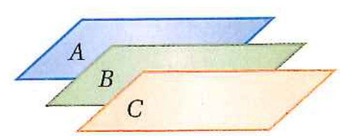
$\overleftrightarrow{AB}$ is the intersection of P and Q.

Exercises

Work cooperatively. Write a description for each figure.

1. 

2. 

3. 

4. Draw and label a figure for the statement: *Planes N and P contain line a.*

LESSON

1-2 Line Segments and Distance

Then

- You identified points, lines, and planes. (Lesson 1-1)

Now

1. Calculate with measures.
2. Find the distance between two points.

Why?

- Dallas, Grand Prairie, and Arlington are cities in Texas. Grand Prairie is between Dallas and Arlington. If you drive from Dallas to Arlington, you will pass through Grand Prairie.

Targeted TEKS

G.2(B) Derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines.

G.5(B) Construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge. *Also addresses G.5(C).*

Mathematical Processes

G.1(E) Create and use representations to organize, record, and communicate mathematical ideas. *Also addresses G.1(B).*

New Vocabulary

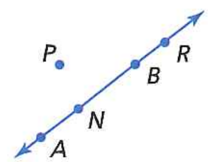
- line segment
- betweenness of points
- between
- congruent
- rigid transformation
- congruent segments
- construction
- distance
- irrational number

1 Calculate Measures

Unlike a line, a **line segment** can be measured because it has two endpoints. These endpoints are used to name the segment.

Meaning	Notation
a segment with endpoints *A* and *B*	$\overline{AB}$ or $\overline{BA}$
a line that contains points *A* and *B*	$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$

You know that for any two real numbers a and b, there is a real number n between a and b such that $a < n < b$. This relationship also applies to points on a line and is called **betweenness of points**. In the figure, point N is between points A and B, but points R and P are not between A and B.

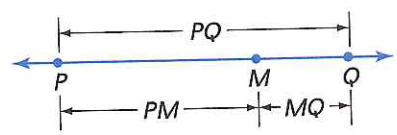


Key Concept Betweenness of Points

Words

Point M is **between** points P and Q if and only if P, Q, and M are collinear and $PM + MQ = PQ$.

Model



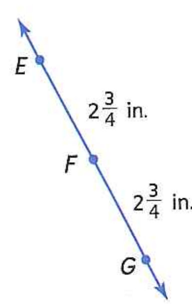
The measure of $\overline{AB}$ is written as AB. Measures are real numbers, so all arithmetic operations can be used with them. You know that the whole equals the sum of its parts. This is also true of line segments in geometry.

Example 1 Finding Measurements by Adding or Subtracting

Find each measure. Assume that the figures are not drawn to scale.

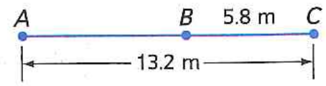
a. EG

EG is the measure of $\overline{EG}$. Point F is between E and G. Find EG by adding EF and FG.



$EF + FG = EG$	Betweenness of points
$2\frac{3}{4} + 2\frac{3}{4} = EG$	Substitution
$5\frac{1}{2}$ in. $= EG$	Add.

b. AB

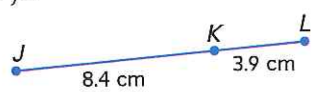


Point B is between points A and C.

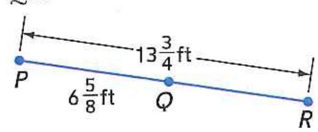
$AB + BC = AC$	Betweenness of points
$AB + 5.8 = 13.2$	Substitution
$AB + 5.8 - 5.8 = 13.2 - 5.8$	Subtract 5.8 from each side.
$AB = 7.4$ m	Simplify.

GuidedPractice

1A. JL



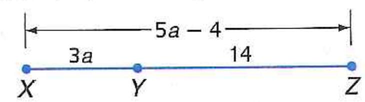
1B. QR



Example 2 Write and Solve Equations to Find Measurements

ALGEBRA Find the value of a and XY if Y is between X and Z, $XY = 3a$, $XZ = 5a - 4$, and $YZ = 14$.

Draw a figure to represent the information.



$XZ = XY + YZ$	Betweenness of points
$5a - 4 = 3a + 14$	Substitution
$5a - 4 - 3a = 3a + 14 - 3a$	Subtract $3a$ from each side.
$2a - 4 = 14$	Simplify.
$2a - 4 + 4 = 14 + 4$	Add 4 to each side.
$2a = 18$	Simplify.
$\frac{2a}{2} = \frac{18}{2}$	Divide each side by 2.
$a = 9$	Simplify.

CHECK

$5a - 4 = 3a + 14$	Original Equation
$5(9) - 4 \stackrel{?}{=} 3(9) + 14$	Substitution
$45 - 4 \stackrel{?}{=} 27 + 14$	Multiply.
$41 = 41$ ✓	Simplify.

Now find XY.

$XY = 3a$	Given
$= 3(9)$ or 27	$a = 9$

GuidedPractice

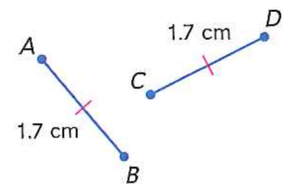
2. Find x and BC if B is between A and C, $AC = 4x - 12$, $AB = x$, and $BC = 2x + 3$.

StudyTip

Equal vs. Congruent Lengths Lengths are *equal* and segments are *congruent*. It is correct to say that $AB = CD$ and $\overline{AB} \cong \overline{CD}$. However, it is not correct to say that $\overline{AB} = \overline{CD}$ or that $AB \cong CD$.

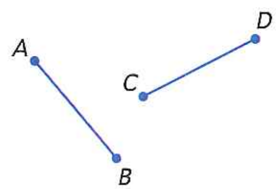
If two geometric figures have exactly the same shape and size, they are **congruent**. Recall that a *transformation* is an operation that maps a geometric figure, the *preimage*, onto a new figure called the *image*. A **rigid transformation** is a transformation in which the position of the image may differ from that of the preimage, but the two figures remain congruent. If one segment can be mapped onto another segment using rigid transformations, then the two segments are called **congruent segments**. If two segments are congruent, then they have the same measure.

KeyConcept Congruent Segments

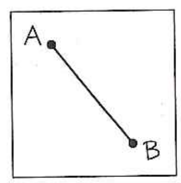
Words	Congruent segments have the same measure.	
Symbols	≅ is read *is congruent to.* Red slashes on the figure also indicate congruence.	
Example	$\overline{AB} \cong \overline{CD}$	
Meaning	Segment *AB* is congruent to segment *CD*.	

Tracing paper or a transparency sheet can be used to verify congruence by rigid transformations.

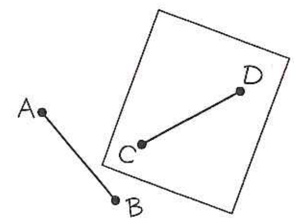
Show that $\overline{AB} \cong \overline{CD}$.



Step 1 Using tracing paper or a transparency sheet, trace $\overline{AB}$.



Step 2 Rotate and slide the transparency to show that $\overline{AB} \cong \overline{CD}$.



Since $\overline{AB}$ maps to $\overline{CD}$ exactly, $\overline{AB} \cong \overline{CD}$.

Drawings of geometric figures are created using measurement tools such as a ruler and a protractor. **Constructions** are methods of creating these figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used in constructions. *Sketches* are created using pencil only.

StudyTip

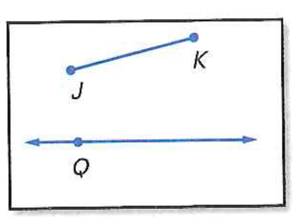
Lines vs. Segments
Remember that a segment has two endpoints, and a line extends indefinitely in both directions.

You can construct a segment by first using a compass to establish the length of the given segment.

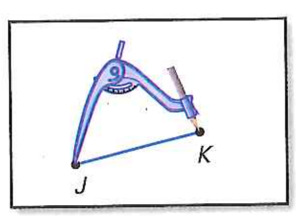
TEKS G.5(B)

Construction Copy a Segment

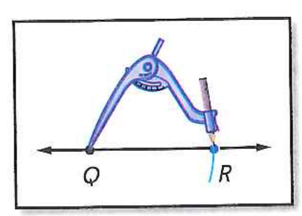
Step 1 Draw a segment $\overline{JK}$. Elsewhere on your paper, draw a line and a point on the line. Label the point *Q*.



Step 2 Place the compass at point *J* and adjust the compass setting so that the pencil is at point *K*.



Step 3 Using that setting, place the compass point at *Q* and draw an arc that intersects the line. Label the point of intersection *R*. $\overline{JK} \cong \overline{QR}$.



2 Distance Between Two Points

The **distance** between two points is the length of the segment with those points as its endpoints. The coordinates of the points can be used to find this length.

StudyTip

Distance Formula The Distance Formula uses absolute values because distances cannot be negative.

KeyConcept Distance Formula (on Number Line)

Words The distance between two points is the absolute value of the difference between their coordinates.

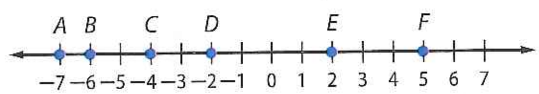
Symbols If P has coordinate x_1 and Q has coordinate x_2, $PQ = |x_2 - x_1|$ or $|x_1 - x_2|$.

Because $\overline{PQ}$ is the same as $\overline{QP}$, the order in which you name the endpoints is not important when calculating distance.

TEKS G.2(B)

Example 3 Find Distance on a Number Line

Use the number line.



a. Find *BE*.

The coordinates of B and E are −6 and 2.

$BE = |x_2 - x_1|$ — Distance Formula

$= |2 - (-6)|$ — $x_1 = -6$ and $x_2 = 2$

$= |8|$ or 8 — Simplify.

b. Determine whether *CA* and *ED* are congruent.

The coordinates of C and A are −4 and −7. Those of E and D are 2 and −2.

CA		ED
$CA = \|x_2 - x_1\|$	Distance Formula	$ED = \|x_2 - x_1\|$
$= \|-7 - (-4)\|$	Substitute.	$= \|-2 - 2\|$
$= \|-3\|$	Subtract.	$= \|-4\|$
$= 3$	Simplify.	$= 4$

No, the segments are not congruent.

GuidedPractice

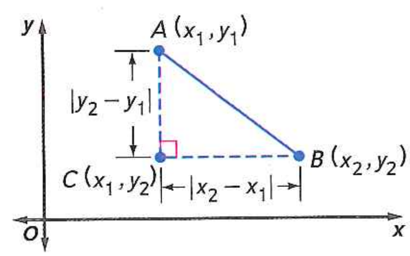
3A. Find AD.

3B. Determine whether CF and AE are congruent.

StudyTip

Pythagorean Theorem The Pythagorean Theorem is often expressed as $a^2 + b^2 = c^2$, where a and b are the measures of the shorter sides (legs) of a right triangle, and c is the measure of the longest side (hypotenuse). You will prove and learn about other applications of the Pythagorean Theorem in Lesson 8-2.

To find the distance between two points A and B in the coordinate plane, you can form a right triangle with $\overline{AB}$ as its hypotenuse and point C as its vertex as shown. Then use the Pythagorean Theorem to find AB.



$(CB)^2 + (AC)^2 = (AB)^2$ — Pythagorean Theorem

$(|x_2 - x_1|)^2 + (|y_2 - y_1|)^2 = (AB)^2$ — $CB = |x_2 - x_1|$, $AC = |y_2 - y_1|$

$(x_2 - x_1)^2 + (y_2 - y_1)^2 = (AB)^2$ — The square of a number is always positive.

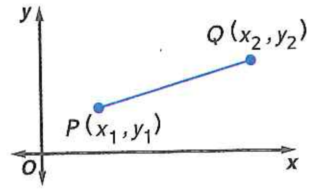
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = AB$ — Take the positive square root of each side.

This gives us the Distance Formula for points in the coordinate plane. Because this formula involves taking the square root of a number, distances can be irrational. Recall that an **irrational number** is a number that cannot be expressed as a terminating or repeating decimal.

KeyConcept Distance Formula (in Coordinate Plane)

If P has coordinates (x_1, y_1) and Q has coordinates (x_2, y_2) then

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The order of the x- and y-coordinates in each set of parentheses is not important.

Example 4 Find Distance on a Coordinate Plane

Find the distance between each pair of points.

a. $C(-4, -6)$ and $D(5, -1)$

$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[5 - (-4)]^2 + [-1 - (-6)]^2}$ $(x_1, y_1) = (-4, -6)$ and $(x_2, y_2) = (5, -1)$

$= \sqrt{9^2 + 5^2}$ or $\sqrt{106}$ Subtract.

The distance between C and D is $\sqrt{106}$ or approximately 10.3 units.

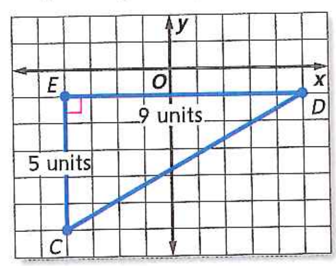
CHECK Graph the ordered pairs and check by using the Pythagorean Theorem.

$(CD)^2 = (EC)^2 + (ED)^2$

$(CD)^2 = 5^2 + 9^2$

$(CD)^2 = 106$

$CD = \sqrt{106}$ ✓



StudyTip

Distance on a Coordinate Plane To find the distance between E and D in Example 4 count the squares on the grid from E to D or find the absolute value of the difference between the x-coordinates, $|-4 - 5| = 9$.

b. $M(2, 4)$ and $N(-3, -2)$

$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[(-3) - 2]^2 + [(-2) - 4]^2}$ $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (-3, -2)$

$= \sqrt{(-5)^2 + (-6)^2}$ or $\sqrt{61}$ Subtract.

The distance between M and N is $\sqrt{61}$ or approximately 7.81 units.

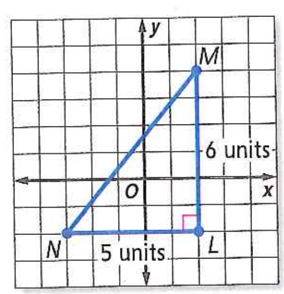
CHECK Graph the ordered pairs and check by using the Pythagorean Theorem.

$(MN)^2 = (LM)^2 + (LN)^2$

$(MN)^2 = 6^2 + 5^2$

$(MN)^2 = 61$

$MN = \sqrt{61}$ ✓



Go Online!

Look for the **Tools** icons for places where the tools in the eToolkit may be useful. Log into ConnectED to use the tools.

GuidedPractice

Find the distance between each pair of points.

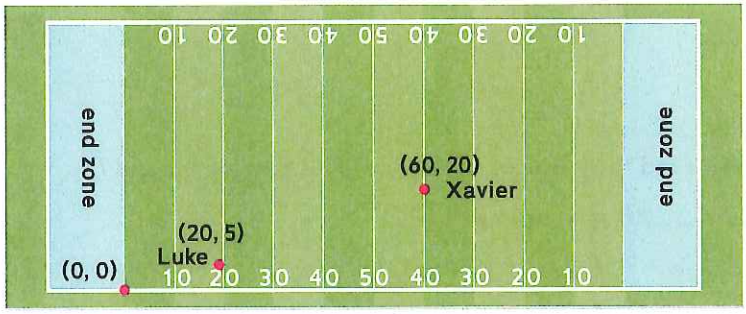
4A. $E(-5, 6)$ and $F(8, -4)$

4B. $J(4, 3)$ and $K(-3, -7)$

Real-World Example 5 Find Distance

FOOTBALL Luke is standing on his team's 20-yard line, 5 yards from the sideline, when he throws the football. Xavier catches it on the other team's 40-yard line, 20 yards from the same sideline. How far did Luke throw the football?

Analyze First draw a diagram to represent the situation.



Place a point where one of the end zones intersects a sideline. Label this point (0, 0). Place a second point on the 20-yard line that is on the same side of the field as the origin and is about 5 yards away from the sideline. Label the point (20, 5). Next, place a third point on the opposite 40 yard line about 20 yards away from the same sideline. Label the point (60, 20) because it is 60 yards away from the origin.

Formulate Use the Distance Formula knowing $(x_1, y_1) = (20, 5)$ and $(x_2, y_2) = (60, 20)$.

Determine

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(60 - 20)^2 + (20 - 5)^2}$	$(x_1, y_1) = (20, 5)$ and $(x_2, y_2) = (60, 20)$
$= \sqrt{40^2 + 15^2}$	Subtract.
$= \sqrt{1600 + 225}$	Square each term.
$= \sqrt{1825}$	Add.
≈ 42.7	Take the positive square root.

Luke threw the football approximately 42.7 yards.

Justify Luke is **40 yards** from the other team's 40-yard line where Xavier is located, and Xavior is **15 yards** further from the sideline than Luke.

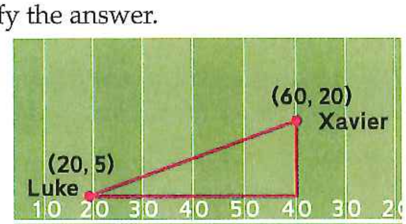
Using the Pythagorean Theorem to justify the answer.

$c^2 = a^2 + b^2$

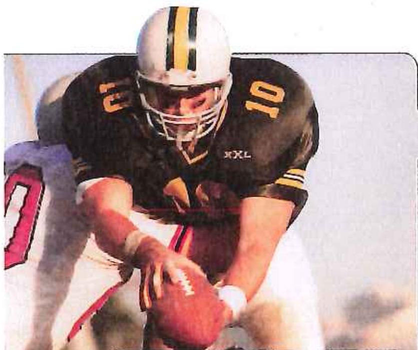
$c^2 = 40^2 + 15^2$

$c^2 = 1825$

$c \approx 42.7$ yards



Evaluate We were able to determine Luke's and Xavier's locations in terms of points on a coordinate plane and we know how to use the Distance Formula to calculate the distance between two points on a coordinate plane. The distance we calculated seems reasonable.



Real-World Link

Some schools are using Fantasy Football to help teach mathematics. Students calculate statistics from that week's games in order to determine their team's score.

Source: ESPN

Guided Practice

5. Using the diagram above, find the distance that Luke threw the football if he was standing on the 30-yard line about 10 yards from the sideline, and Xavier was standing at the 50-yard line about 40 yards from the same sideline.

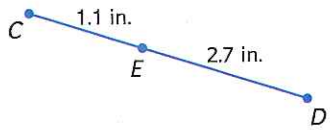
Check Your Understanding

= Step-by-Step Solutions begin on page R14.

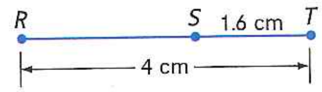
Go Online! for a Self-Check Quiz

Example 1 **Find each measure. Assume that the figures are not drawn to scale.**

1. CD



2. RS

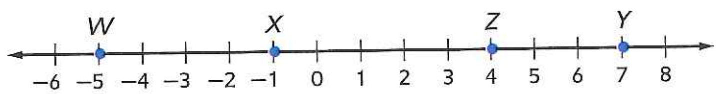


Example 2 **ALGEBRA** **Find the value of x and BC if B is between C and D.**

3. $CB = 2x$, $BD = 4x$, and $BD = 12$

4. $CB = 4x - 9$, $BD = 3x + 5$, and $CD = 17$

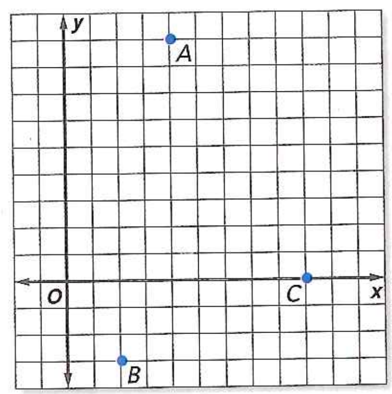
Example 3 TEKS G.2(B) **Use the number line to find each measure.**



5. XY

6. WZ

Example 4 **TIME CAPSULE** Graduating classes have buried time capsules on the campus of East Side High School for over twenty years. The points on the diagram show the position of three time capsules. Find the distance between each pair of time capsules.



7. $A(4, 9)$, $B(2, -3)$

8. $A(4, 9)$, $C(9, 0)$

9. $B(2, -3)$, $C(9, 0)$

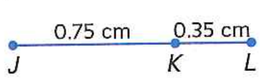
Example 5 10. MP **ANALYZE RELATIONSHIPS** Which two time capsules are the closest to each other? Which are farthest apart?

Practice and Problem Solving

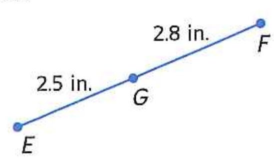
Extra Practice is found on page R1.

Example 1 **Find each measure. Assume that the figures are not drawn to scale.**

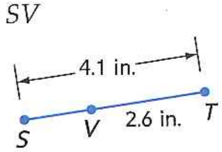
11. JL



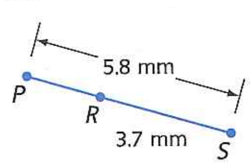
12. EF



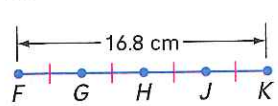
13. SV



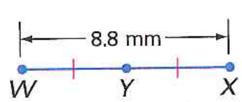
14. PR



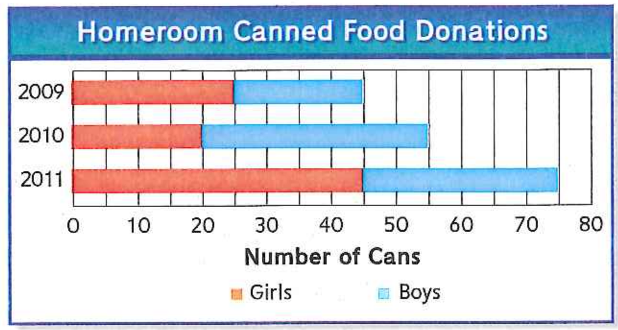
15. FG



16. WY



17. MP **ORGANIZE IDEAS** The stacked bar graph shows the number of canned food items donated by the students in a homeroom class over three years. Use the concept of betweenness of points to find the number of cans donated by the boys for each year. Explain your method.



Example 2 **ALGEBRA** **Find the value of the variable and YZ if Y is between X and Z.**

18. $XY = 11, YZ = 4c, XZ = 83$

19. $XY = 6b, YZ = 8b, XZ = 175$

20. $XY = 7a, YZ = 5a, XZ = 6a + 24$

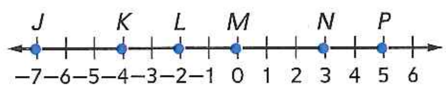
21. $XY = 11d, YZ = 9d - 2, XZ = 5d + 28$

22. $XY = 4n + 3, YZ = 2n - 7, XZ = 22$

23. $XY = 3a - 4, YZ = 6a + 2, XZ = 5a + 22$

Example 3
TEKS G.2(B)

Use the number line to find each measure.



24. JL

25. JK

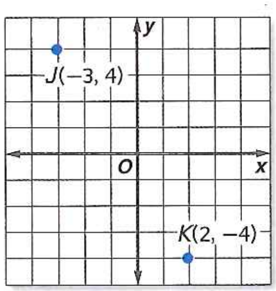
26. KP

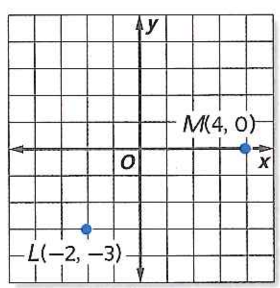
27. NP

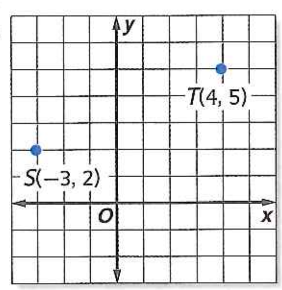
28. JP

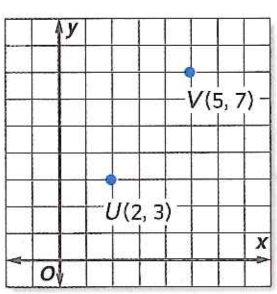
29. LN

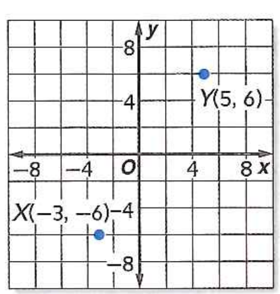
Example 4 **Find the distance between each pair of points.**

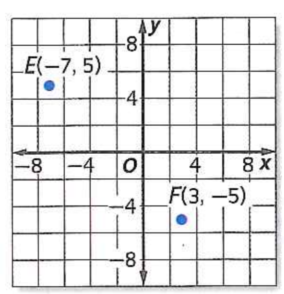
30.


31.


32.


33.


34.


35.


Find the distance between each pair of points.

36. $X(1, 4), Y(6, 9)$

37. $P(3, 4), Q(7, 2)$

38. $M(-3, 8), N(-5, 1)$

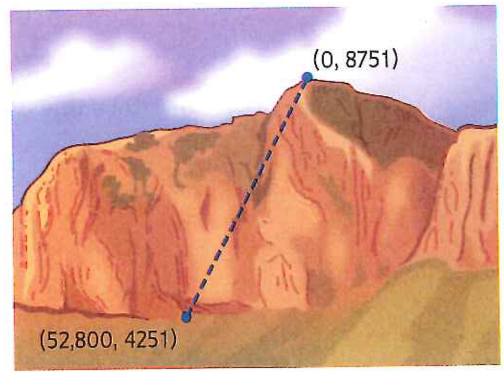
39. $Y(-4, 9), Z(-5, 3)$

40. $A(2, 4), B(5, 7)$

41. $C(5, 1), D(3, 6)$

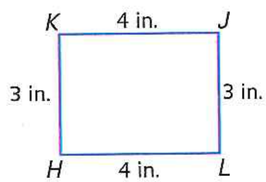
Example 5

42. MP **PROBLEM SOLVING** Vivian is planning to hike to the top of Guadalupe Peak on her family vacation. The coordinates of the peak of the mountain and of the base of the trail are shown in feet. If the trail can be approximated by a straight line, estimate the length of the trail to the nearest tenth of a mile. (*Hint:* 1 mi = 5280 ft)

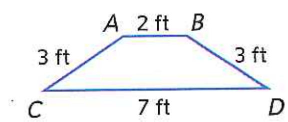


Determine whether each pair of segments is congruent.

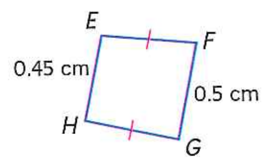
43. $\overline{KJ}, \overline{HL}$



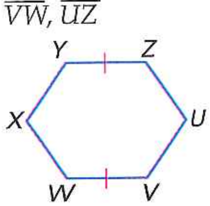
44. $\overline{AC}, \overline{BD}$



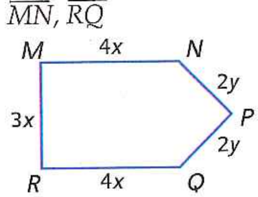
45. $\overline{EH}, \overline{FG}$



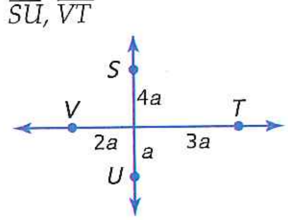
46. $\overline{VW}, \overline{UZ}$



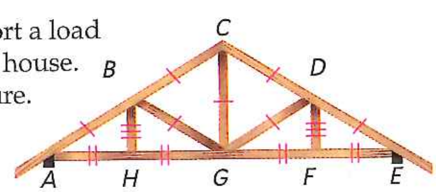
47. $\overline{MN}, \overline{RQ}$



48. $\overline{SU}, \overline{VT}$



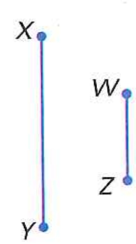
49. **TRUSSES** A truss is a structure used to support a load over a span, such as a bridge or the roof of a house. List all of the congruent segments in the figure.



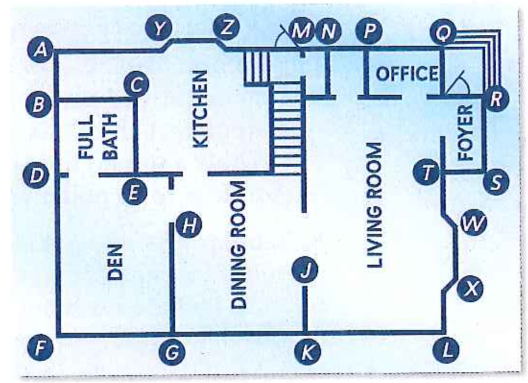
50. **CONSTRUCTION** For each expression:

- construct a segment with the given measure,
- explain the process you used to construct the segment, and
- verify that the segment you constructed has the given measure.

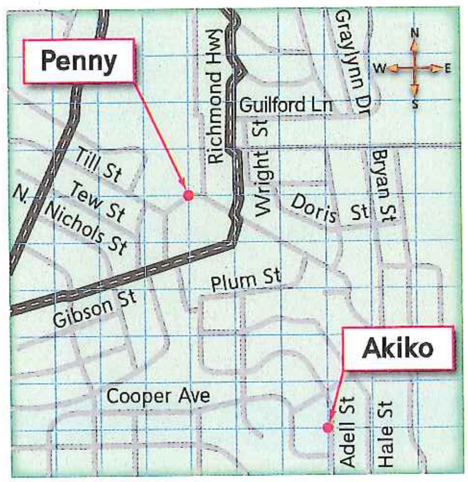
a. WZ
b. $2(XY)$
c. $6(WZ) - XY$



51 **BLUEPRINTS** Use a ruler to determine at least five pairs of congruent segments with labeled endpoints in the blueprint shown.



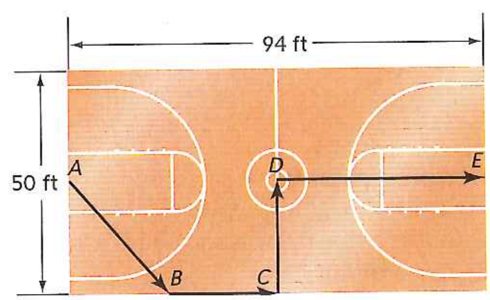
52. MP **APPLY MATH** Penny and Akiko live in the locations shown on the map below.



a. If each square on the grid represents one block and the bottom left corner of the grid is the location of the origin, what is the distance from Penny's house to Akiko's?

b. If Penny moves three blocks to the north and Akiko moves 5 blocks to the west, how far apart will they be?

53. **MULTI-STEP** Coach Willis designs a play that requires the ball to be passed from point A to point E as shown below. The arrows represent quick passes to different members of his team. Randi can throw the ball from under the basket to midcourt, Jen and Mandy can throw the ball half the width of the court, Makayla can throw the ball to the free throw line from under the basket, and Kim can throw the ball farther than Jen.



a. In which position should each girl be?

b. Describe your solution process.

c. What assumptions did you make?

54. MULTIPLE REPRESENTATIONS Betweenness of points ensures that a line segment may be divided into an infinite number of line segments.

a. **Geometric** Use a ruler to draw a line segment 3 centimeters long. Label the endpoints A and D. Draw two more points along the segment and label them B and C. Draw a second line segment 6 centimeters long. Label the endpoints K and P. Add four more points along the line and label them L, M, N, and O.

b. **Tabular** Use a ruler to measure the length of the line segment between each of the points you have drawn. Organize the lengths of the segments in $\overline{AD}$ and $\overline{KP}$ into a table. Include a column in your table to record the sum of these measures.

$\overline{AD}$		$\overline{KP}$	
Segment	Length (cm)	Segment	Length (cm)
$\overline{AB}$		$\overline{KL}$	
$\overline{BC}$		$\overline{LM}$	
$\overline{CD}$		$\overline{MN}$	
Total		$\overline{NO}$	
		$\overline{OP}$	
		Total	

c. **Algebraic** Write an equation that could be used to find the lengths of $\overline{AD}$ and $\overline{KP}$. Compare the lengths determined by your equation to the actual lengths.

TEKS G.2(B), G.5(B), G.5(C)

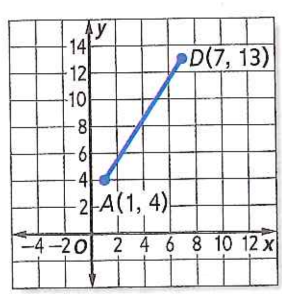
H.O.T. Problems Use Higher-Order Thinking Skills

55. WRITING IN MATH If point B is between points A and C, explain how you can find AC if you know AB and BC. Explain how you can find BC if you know AB and AC.

56. TOOLS AND TECHNIQUES Draw a segment $\overline{AB}$ that measures between 2 and 3 inches long. Then sketch a segment $\overline{CD}$ congruent to $\overline{AB}$, draw a segment $\overline{EF}$ congruent to $\overline{AB}$, and construct a segment $\overline{GH}$ congruent to $\overline{AB}$. Compare your methods.

57. JUSTIFY ARGUMENTS Determine whether the statement *If point M is between points C and D, then CD is greater than either CM or MD* is *sometimes, never,* or *always* true. Explain.

58. ANALYZE RELATIONSHIPS Point P is located on the segment between point $A(1, 4)$ and point $D(7, 13)$. The distance from A to P is twice the distance from P to D. What are the coordinates of point P?

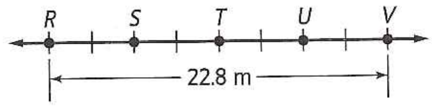


59. WRITING IN MATH Explain how the Pythagorean Theorem is used to derive the Distance Formula.

Preparing for Assessment

Example TEKS G.2(B) MP G.1(C), G.1(F)

TEKS REVIEW What is the length of $\overline{RU}$?



A 5.7 m

B 11.4 m

C 17.1 m

D 22.8 m

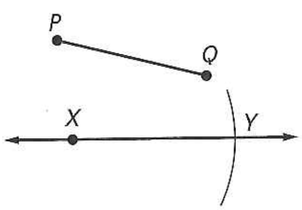
The figure shows that the four segments $\overline{RS}$, $\overline{ST}$, $\overline{TU}$, and $\overline{UV}$ are congruent. Therefore, $\overline{RV}$ is divided into four segments of equal length.

$$\begin{aligned} RS &= \tfrac{1}{4}(RV) \\ &= \tfrac{1}{4}(22.8) \\ &= 5.7 \text{ m} \end{aligned}$$

$$\begin{aligned} RU &= 3(RS) \\ &= 3(5.7) \text{ or } 17.1 \text{ m} \end{aligned}$$

The length of $\overline{RU}$ is 17.1 m. The correct answer is choice C.

60. Jonah draws $\overline{PQ}$. Then he draws a line and labels point X on the line. Next, he places the compass point at P and opens the compass so the pencil is at point Q. Using that setting, he places the compass point at X and draws an arc that intersects the line at point Y.



Which of the following must be true? TEKS G.5(B) MP G.1(D), G.1(F)

I. $PQ = XY$

II. $\overline{PQ} \cong \overline{XY}$

III. $PX = QY$

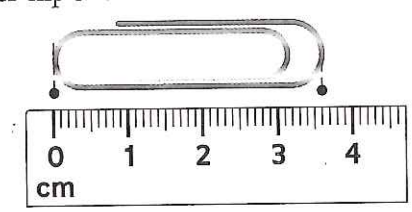
A I only

B II only

C I and II only

D I, II, and III

61. Shauntay has a thumbtack that is $\frac{1}{3}$ as long as the paper clip shown below.



Which of the following is the length of the thumbtack? TEKS G.2(A) MP G.1(A), G.1(F)

F 1.2 cm

G 10.8 cm

H 3.6 cm

J 0.33 cm

62. Isaac draws $\overline{MN}$ so that $MN = 5.2$ cm. Then he draws a point P between M and N so that P is $\frac{1}{4}$ of the distance from M to N.

Which of the following is the length of $\overline{PN}$? TEKS G.2(A) MP G.1(E), G.1(F)

A 1.3 cm

B 3.9 cm

C 20.8 cm

D 15.6 cm

LESSON

1-3 Locating Points and Midpoints

∵ Then	∵ Now	∵ Why?
You found the distance between two points on a line segment.	1 Find the midpoint of a segment. 2 Locate a point on a segment a given fractional distance from one endpoint.	Adrian, Texas, is located on historic Route 66. Adrian is famous for being the midpoint of Route 66, 1139 miles from its endpoints, Los Angeles and Chicago.

TEKS Targeted TEKS

G.2(A) Determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint.

G.2(B) Derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines.

Also addresses G.5(B).

MP Mathematical Processes

G.1(A) Apply mathematics to problems arising in everyday life, society, and the workplace.

G.1(G) Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

NewVocabulary

midpoint
segment bisector

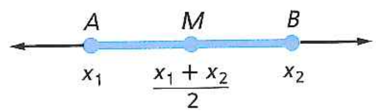
1 Midpoint of a Segment

The **midpoint** of a segment is the point halfway between the endpoints of the segment. If M is the midpoint of $\overline{AB}$, then $AM = MB$ and $\overline{AM} \cong \overline{MB}$. To find the midpoint of a segment on a number line, find the *mean*, or average, of the coordinates of its endpoints.

KeyConcept Midpoint Formula (on Number Line)

If $\overline{AB}$ has endpoints at x_1 and x_2 on a number line, then the midpoint M of $\overline{AB}$ has coordinate

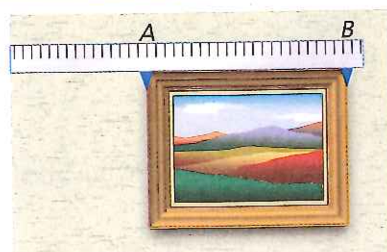
$$\frac{x_1 + x_2}{2}.$$



TEKS G.2(A), G.2(B)

Real-World Example 1 Find the Midpoint on a Number Line

DECORATING Jacinta is hanging a picture 15 inches from the left side of a wall. How far from the edge of the wall should she mark the location for the nail the picture will hang on if the right edge is 37.5 inches from the wall's left side?



The coordinates of the endpoints of the top of the picture frame are 15 inches and 37.5 inches. Let M be the midpoint of $\overline{AB}$.

$M = \dfrac{x_1 + x_2}{2}$ Midpoint Formula

$= \dfrac{15 + 37.5}{2}$ $x_1 = 15, x_2 = 37.5$

$= \dfrac{52.5}{2}$ or 26.25 Simplify.

The midpoint is located 26.25 inches from the left edge of the wall.

GuidedPractice

1. **TEMPERATURE** The temperature on a thermometer dropped from a reading of 25° to −8°. Find the midpoint of these temperatures.

The x-coordinate of the midpoint of a segment on the coordinate plane is the average of the x-coordinates of the endpoints of the segment. Similarly, the y-coordinate of the midpoint of a segment is the average of the y-coordinates of the endpoints.

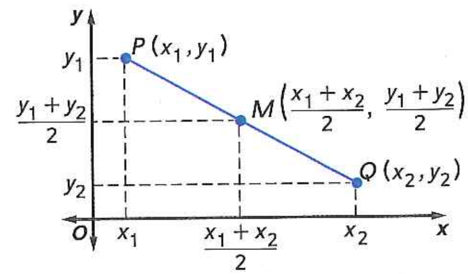
StudyTip

Using Your Text Look for Key Concepts to learn important properties, definitions, and concepts.

KeyConcept Midpoint Formula (in Coordinate Plane)

If $\overline{PQ}$ has endpoints at $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane, then the midpoint M of $\overline{PQ}$ has coordinates

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



When finding the midpoint of a segment, the order of the endpoints is not important.

TEKS G.2(A), G.2(B)

Example 2 Find the Midpoint in the Coordinate Plane

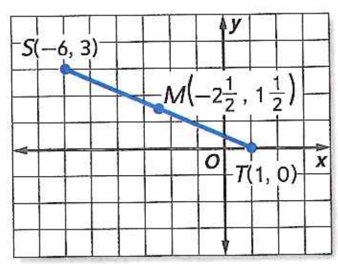
Find the coordinates of M, the midpoint of $\overline{ST}$, for $S(-6, 3)$ and $T(1, 0)$.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint Formula

$= \left(\frac{-6 + 1}{2}, \frac{3 + 0}{2}\right)$ $(x_1, y_1) = S(-6, 3), (x_2, y_2) = T(1, 0)$

$= \left(\frac{-5}{2}, \frac{3}{2}\right)$ or $\left(-2\frac{1}{2}, 1\frac{1}{2}\right)$ Simplify.

CHECK Graph S, T, and M. The distance from S to M does appear to be the same as the distance from M to T, so our answer is reasonable.



StudyTip

Comparing Measures Another way to check your answer is to use the Distance Formula to compare the distances between S and M and M and T.

GuidedPractice

Find the coordinates of the midpoint of a segment with the given coordinates.

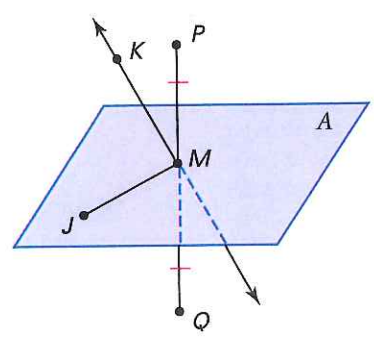
2A. $A(5, 12)$, $B(-4, 8)$

2B. $C(-8, -2)$, $D(5, 1)$

StudyTip

Segment Bisectors Each segment has an infinite number of bisectors. Each bisector must contain the midpoint of the segment.

Any segment, line, or plane that intersects a segment at its midpoint is called a **segment bisector**. In the figure, M is the midpoint of $\overline{PQ}$. Plane A, $\overline{MJ}$, $\overleftrightarrow{KM}$, and point M are all bisectors of $\overline{PQ}$. We say that they each bisect $\overline{PQ}$.

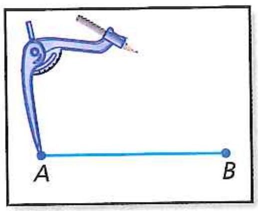


The construction shows how to construct a line that bisects a segment.

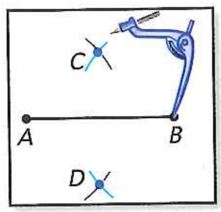
TEKS G.5(B)

Construction Bisect a Segment

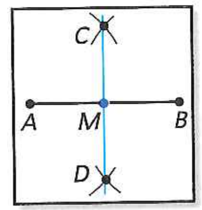
Step 1 Draw a segment and name it $\overline{AB}$. Place the compass at point A. Adjust the compass so that its width is greater than $\frac{1}{2}AB$. Draw arcs above and below $\overline{AB}$.



Step 2 Using the same compass setting, place the compass at point B and draw arcs above and below $\overline{AB}$ so that they intersect the two arcs previously drawn. Label the points of the intersection of the arcs as C and D.



Step 3 Use a straightedge to draw $\overline{CD}$. Label the point where it intersects $\overline{AB}$ as M. Point M is the midpoint of $\overline{AB}$, and $\overline{CD}$ is a bisector of $\overline{AB}$.



If you know that a given point is the midpoint of a segment, then you can use algebra and other key information to find missing measures or coordinates.

TEKS G.2(B)

Example 3 Find Missing Coordinates

Find the coordinates of J if $K(-1, 2)$ is the midpoint of $\overline{JL}$ and L has the coordinates $(3, -5)$.

Step 1 Substitute the information you are given into the Midpoint Formula. Let J be (x_1, y_1) and L be (x_2, y_2).

$$\left(\frac{x_1 + 3}{2}, \frac{y_1 + (-5)}{2}\right) = (-1, 2) \qquad (x_2, y_2) = (3, -5)$$

Step 2 Write two equations to find the coordinates of J.

$$\frac{x_1 + 3}{2} = -1 \qquad \text{Midpoint Formula}$$

$$x_1 + 3 = -2 \qquad \text{Multiply each side by 2.}$$

$$x_1 = -5 \qquad \text{Subtract 3 from each side.}$$

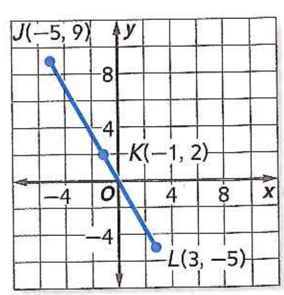
$$\frac{y_1 + (-5)}{2} = 2 \qquad \text{Midpoint Formula}$$

$$y_1 - 5 = 4 \qquad \text{Multiply each side by 2.}$$

$$y_1 = 9 \qquad \text{Add 5 to each side.}$$

The coordinates of J are $(-5, 9)$.

CHECK Graph J, K, and L. The distance from J to K does appear to be the same as the distance from K to L, so our answer is reasonable.



StudyTip

Check for Reasonableness Always graph the given information and the calculated coordinates of the third point to check the reasonableness of your answer.

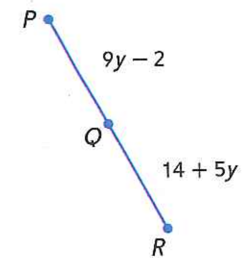
Guided Practice

3. Find the coordinates of G if P is the midpoint of $\overline{EG}$ for $E(-8, 6)$ and $P(-5, 10)$

TEKS G.2(B)

Example 4 Find Missing Measures

Find the measure of $\overline{PQ}$ if Q is the midpoint of $\overline{PR}$.



Because Q is the midpoint, you know that $PQ = QR$. Use this equation to find the value of y.

$PQ = QR$	Definition of midpoint
$9y - 2 = 14 + 5y$	$PQ = 9y - 2$, $QR = 14 + 5y$
$4y - 2 = 14$	Subtract $5y$ from each side.
$4y = 16$	Add 2 to each side.
$y = 4$	Divide each side by 4.

Now substitute 4 for y in the expression for PQ.

$PQ = 9y - 2$	Given
$= 9(4) - 2$	$y = 4$
$= 36 - 2$ or 34	Simplify.

The measure of $\overline{PQ}$ is 34.

Guided Practice

4. Find the value of x if C is the midpoint of $\overline{AB}$, $AC = 4x + 5$, and $AB = 78$.

2 Locate Points

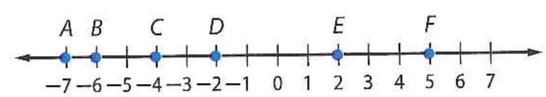
You can think of the midpoint of a segment as being half the distance from one endpoint to the other. You can also find points that are other fractional distances from an endpoint.

TEKS G.2(A)

Example 5 Locating a Point at Fractional Distances

Find X on $\overline{AF}$ that is $\frac{1}{6}$ of the distance from A to F.

Find the distance between A and F.



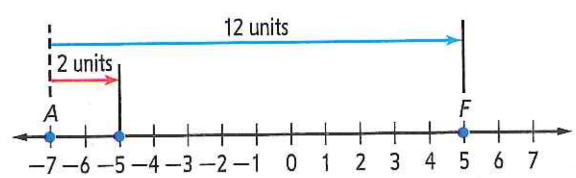
$AF = \lvert x_2 - x_1 \rvert$	Distance Formula
$= \lvert 5 - (-7) \rvert$	$x_1 = -7$ and $x_2 = 5$
$= \lvert 12 \rvert$ or 12	Subtract.

The distance between A and F is 12 units.

To find the point $\frac{1}{6}$ of the distance from A to F find $\frac{1}{6}AF$.

$12\left(\frac{1}{6}\right) = 2$

X is 2 units from point A on $\overline{AF}$ so, point X is located at $-7 + 2 = -5$ on the number line.



> **Study Tip**
> **Direction** In Example 5, the phrase "from A to F" indicates the direction of the line segment. A is the starting point.

Guided Practice

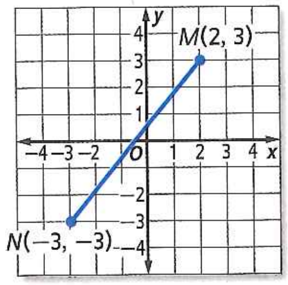
5. Use the number line above to find the point on $\overline{CE}$ that is $\frac{1}{8}$ of the distance from C to E.

You can also find a point a fractional distance from an endpoint on the coordinate plane.

TEKS G.2(A)

Example 6 Fractional Distances on a Coordinate Plane

Find R on $\overline{NM}$ that is $\frac{1}{4}$ the distance from N to M.



Find the distance between the x-coordinates of N and M.

$|x_2 - x_1| = |2 - (-3)|$ $\quad$ $x_1 = -3, x_2 = 2$

$= |5|$ or 5 $\quad$ Subtract.

Multiply this distance by the fractional distance.

$5\left(\frac{1}{4}\right) = 1.25$ Add this to the x-coordinate of N to determine the x-coordinate of R.

$-3 + 1.25 = -1.75$ $\quad$ The x-coordinate of R is -1.75.

Then find the distance between the y-coordinates of N and M.

$|y_2 - y_1| = |3 - (-3)|$ $\quad$ $y_1 = -3, y_2 = 3$

$= |6|$ or 6 $\quad$ Subtract

Next multiply by the fractional distance $\frac{1}{4}$ to get $6\left(\frac{1}{4}\right) = 1.5$. Add this to the y-coordinate of N to find the y-coordinate of R.

$-3 + 1.5 = -1.5$ $\quad$ The y-coordinate of R is -1.5.

The point R is located at $(-1.75, -1.5)$.

GuidedPractice

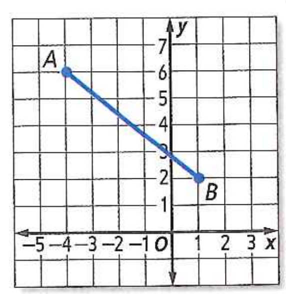
6. Find P on $\overline{NM}$ that is $\frac{1}{5}$ the distance from N to M.

A line segment can also be separated into two or more segments with lengths in a given ratio.

TEKS G.2(A)

Example 7 Locating a Point Given a Ratio

Find C on $\overline{AB}$ such that the ratio of AC to CB is 1:4.



Since the ratio of the measures is 1:4, $4AC = CB$. So, $AB = AC + CB = AC + 4AC$ or $5AC$. Thus, AC is $\frac{1}{5}$ of AB.

Knowing the fractional distance, we can solve as we did in Example 6.

$|x_2 - x_1| = |1 - (-4)| = |5|$ or 5 $\quad$ $x_1 = -4, x_2 = 1$

$|y_2 - y_1| = |2 - 6| = |-4|$ or 4 $\quad$ $y_1 = 6, y_2 = 2$

The distance from the x-coordinate is $5\left(\frac{1}{5}\right) = 1$ and the distance from the y-coordinate is $4\left(\frac{1}{5}\right) = 0.8$. So, C is located at $(-4 + 1, 6 - 0.8)$ or $(-3, 5.2)$.

GuidedPractice

7. Find D on $\overline{AB}$ such that the ratio of AD to DB is 1:3.

Go Online!

Personal Tutors for each example let you follow along as a teacher solves a problem. Pause and rewind as you need.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

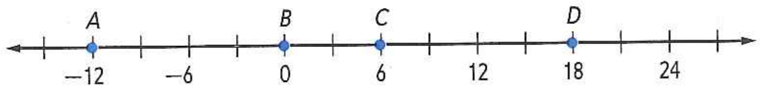
Go Online! for a Self-Check Quiz

Example 1 TEKS G.2(A), G.2(B)

1. **RAINFALL** Mrs. Smith's class measured the amount of rainfall each month during the school year. If the minimum rainfall was 1.7 centimeters and the maximum rainfall was 6.9 centimeters, find the midpoint.

Example 2 TEKS G.2(A), G.2(B)

Use the number line to find the coordinate of the midpoint of each segment.



2. $\overline{BD}$

3. $\overline{AC}$

Find the coordinates of the midpoint of a segment with the given endpoints.

4. $M(7, 1), N(4, -1)$

5. $J(5, -3), K(3, -8)$

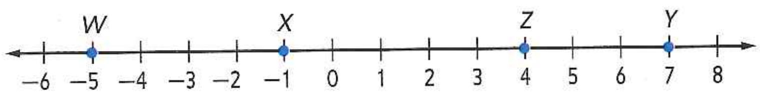
Examples 3–4 TEKS G.2(B)

6. Find the measure of $\overline{LM}$ if M is the midpoint of $\overline{LN}$ and $LM = 3x - 2$ and $MN = 2x + 1$.

7. Find the coordinates of G if $F(1, 3.5)$ is the midpoint of $\overline{GJ}$ and J has coordinates $(6, -2)$.

Example 5 TEKS G.2(A)

Refer to the number line.



8. Find the point N on $\overline{WZ}$ that is $\frac{1}{3}$ of the distance from W to Z.

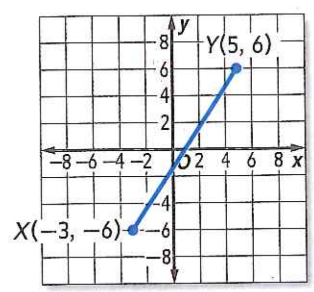
9. Find the point M on $\overline{XY}$ that is $\frac{1}{4}$ of the distance from X to Y.

For Exercises 10 and 11, refer to the coordinate grid shown.

Examples 6–7 TEKS G.2(A)

10. Find M on $\overline{XY}$ that is $\frac{1}{3}$ the distance from X to Y.

11. Find N on $\overline{XY}$ such that the ratio of XN to NY is 1:3.

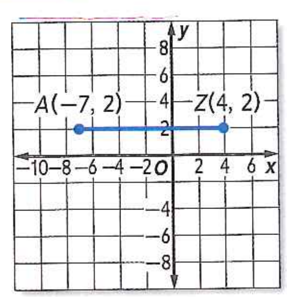


Practice and Problem Solving

Extra Practice is on page R1.

Example 1 TEKS G.2(A), G.2(B)

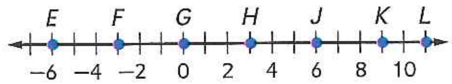
12. **RACE** A city hosts a race annually. They decide to place a camera halfway between the start and finish lines. If A represents the starting line and Z represents the finish line, find the coordinate of the midpoint.



13. **MP APPLY MATH** A business is trying to decide where to build an office. The business wants to place the office halfway between city B and city C. If city B is at (3, 9) and city C is at (3, −5), find the coordinates of the midpoint.

Example 2
TEKS G.2(A), G.2(B)

Use the number line to find the coordinate of the midpoint of each segment.



14. $\overline{JL}$ **15.** $\overline{HK}$ **16.** $\overline{FG}$

17. $\overline{EF}$ **18.** $\overline{EL}$ **19.** $\overline{FK}$

Find the coordinates of the midpoint of a segment with the given endpoints.

20. $W(12, 2), X(7, 9)$

21 $C(22, 4), B(15, 7)$

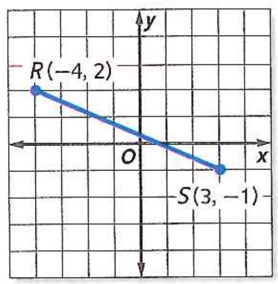
22. $V(-2, 5), Z(3, -17)$

23. $D(-15, 4), E(2, -10)$

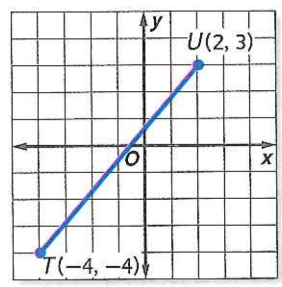
24. $J(-11.2, -3.4), K(-5.6, -7.8)$

25. $X(-2.4, -14), Y(-6, -6.8)$

26.



27.



Example 3
TEKS G.2(B)

Find the coordinates of the missing endpoint if B is the midpoint of $\overline{AC}$.

28. $A(1, 7), B(-3, 1)$ **29.** $C(-5, 4), B(-2, 5)$ **30.** $C(-6, -2), B(-3, -5)$

31. $A(-4, 2), B(6, -1)$ **32.** $C\left(\frac{5}{3}, -6\right), B\left(\frac{8}{3}, 4\right)$ **33.** $A(4, -0.25), B(-4, 6.5)$

Example 4
TEKS G.2(B)

Suppose M is the midpoint of $\overline{FG}$. Find each missing measure.

34. $FM = 5y + 13, MG = 5 - 3y, FG = ?$

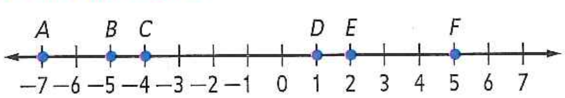
35. $FM = 3x - 4, MG = 5x - 26, FG = ?$

36. $FM = 8a + 1, FG = 42, a = ?$

37. $MG = 7x - 15, FG = 33, x = ?$

Example 5
TEKS G.2(A)

MP **ANALYZE RELATIONSHIPS** **Refer to the number line.**



38. Find the point X on $\overline{CF}$ that is $\frac{1}{5}$ of the distance from C to F.

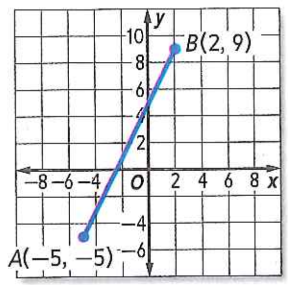
39. Find the point X on $\overline{BD}$ that is $\frac{2}{3}$ of the distance from B to D.

40. Find the point X on $\overline{AE}$ that is $\frac{1}{6}$ of the distance from A to E.

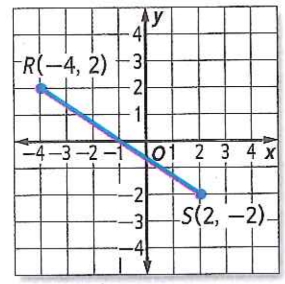
41. Find the point X on $\overline{AF}$ that is $\frac{4}{5}$ of the distance from A to F.

Examples 6–7
TEKS G.2(A)

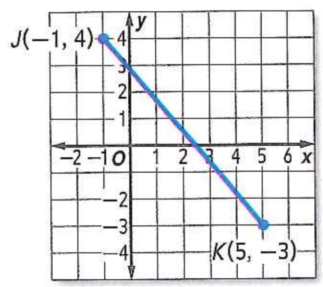
42. Find X on $\overline{AB}$ that is $\frac{1}{5}$ the distance from A to B.



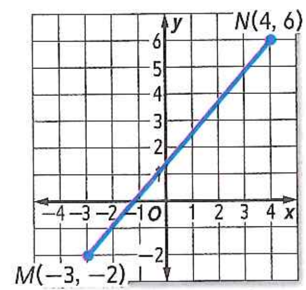
43. Find X on $\overline{RS}$ that is $\frac{1}{6}$ the distance from R to S.



44. Find X on $\overline{JK}$ such that the ratio of JX to XK is 1:2.



45. Find X on $\overline{MN}$ such that the ratio of MX to XN is 2:1.

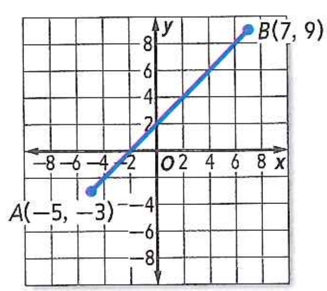


Determine the coordinates of the points that satisfy each condition.

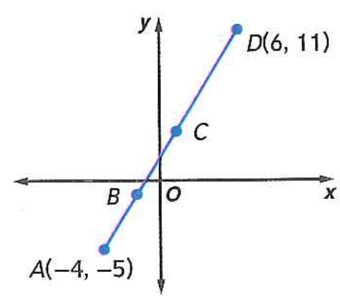
46. Two points on the y-axis are 25 units from (−24, 3).

47 Two points on the x-axis are 10 units from (1, 8).

48. MP **APPLY MATH** Points A and B represent two cities. Where should the state place a rest area so it is halfway between cities A and B?

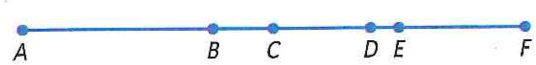


49. **COORDINATE GEOMETRY** Find the coordinates of B if B is halfway between $\overline{AC}$ and C is halfway between $\overline{AD}$.



50. **GEOMETRY** One endpoint of $\overline{AB}$ has coordinates (−3, 5). If the coordinates of the midpoint of $\overline{AB}$ are (2, −6), what is the length of $\overline{AB}$?

51. **CONSTRUCTION** Copy the figure. Use a compass and straightedge to determine whether B is the midpoint of $\overline{AD}$. Explain.



52. **GEOGRAPHY** Refer to the map of Texas. The longitude and latitude of Lubbock and Laredo are shown.

a. Find the latitude and longitude of the midpoint of the segment between Lubbock and Laredo.

b. Use an atlas or the Internet to find a city near the location of the midpoint.

c. If Lubbock is the midpoint of the segment with one endpoint at Laredo, find the latitude and longitude of the other endpoint.

d. Use an atlas or the Internet to find a city near the location of the other endpoint.

e. Find the latitude and longitude of a city $\frac{2}{3}$ of the distance from Laredo to Lubbock.

53. MP **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the midpoint of a segment and the midpoint of the segment between an endpoint and the midpoint.

a. Geometric Use a straightedge to draw three different line segments. Label each pair of endpoints A and B.

b. Geometric For each line segment, use a compass and straightedge to find the midpoint of $\overline{AB}$ and label it C. Then find the midpoint of $\overline{AC}$ and label it D.

c. Tabular Measure and record AB, AC, and AD for each line segment. Organize your results into a table.

d. Algebraic If $AB = x$, write an expression for the measures AC and AD.

e. Verbal Make a conjecture about the relationship between AB and each segment if you were to continue to find the midpoints of a segment and a midpoint you previously found.

54. **MULTI-STEP** John wants to center a canvas, which is 8 feet wide, on his living room wall, which is 17 feet wide. Where on the wall should John mark the location of the nails, if the canvas requires nails every $\frac{1}{5}$ of its length, excluding the edges? Explain your solution process.

TEKS G.2(A), G.5(B)

H.O.T. Problems Use Higher-Order Thinking Skills

55. MP **JUSTIFY ARGUMENTS** Is the point one third of the way from (x_1, y_1) to (x_2, y_2) *sometimes*, *always*, or *never* the point $\left(\frac{x_1 + x_2}{3}, \frac{y_1 + y_2}{3}\right)$? Explain.

56. MP **ANALYZE RELATIONSHIPS** Point P is located on the segment between point $A(1, 4)$ and point $D(7, 13)$. The distance from A to P is twice the distance from P to D. What are the coordinates of point P?

57. MP **ORGANIZE IDEAS** Draw a segment and label it $\overline{AB}$. Using only a compass and a straightedge, construct a segment $\overline{CD}$ such that $CD = 5\frac{1}{4}AB$. Explain and then justify your construction.

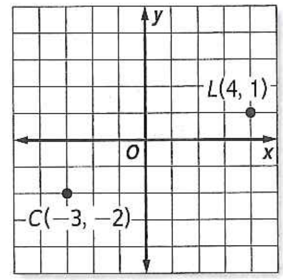
58. **WRITING IN MATH** Describe a method of finding the midpoint of a segment that has one endpoint at (0, 0). Derive the midpoint formula, give an example using your method, and explain why your method works.

Preparing for Assessment

Example

TEKS G.2(B) MP G.1(A), G.1(F)

TEKS REVIEW Each unit of the coordinate plane represents 1 mile. Kate drives along a straight road from city hall, C, to the library, L. Which statement best describes the distance Kate drives?



A Kate drives less than 8 miles.

B Kate drives exactly 10 miles.

C Kate drives exactly 58 miles.

D Kate drives more than 58 miles.

Find the distance between $C(-3, -2)$ and $L(4, 1)$.

$CL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance Formula

$= \sqrt{[4 - (-3)^2] + [1 - (-2)^2]}$ — $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (4, 1)$

$= \sqrt{7^2 + 3^2}$ or $\sqrt{58}$ — Simplify.

Since $\sqrt{58}$ is approximately 7.6, Kate drives less than 8 miles. The correct answer is choice A.

59. Jamar plots two points, P and Q, on a coordinate plane. The midpoint of the points is $M(-3, 4)$.

Which of the following could be the points that Jamar plots? TEKS G.2(B) MP G.1(D), G.1(F)

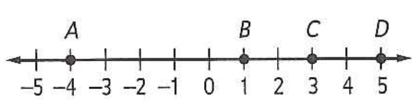
A $P(-5, 10)$ and $Q(1, 2)$

B $P(-2, 6)$ and $Q(-4, 2)$

C $P(-7, 1)$ and $Q(4, 3)$

D $P(-1, 7)$ and $Q(2, 3)$

60. Points A, B, C, and D are located on a number line, as shown.



Which of the following is the distance from the midpoint of $\overline{AB}$ to the midpoint of $\overline{CD}$?
TEKS G.2(B) MP G.1(D), G.1(F)

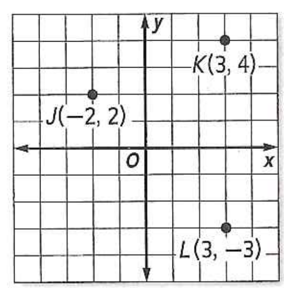
F $2\frac{1}{2}$

G $1\frac{1}{4}$

H $6\frac{1}{2}$

J $5\frac{1}{2}$

61. **ACT/SAT** A video game designer places towers at points J, K, and L, as shown. She plans to place a straight bridge between each pair of towers.



Which of the following is the best estimate of the total length of the bridges?

A 14.1 units

B 12.5 units

C 13.6 units

D 19.5 units

E 128 units

62. Which of the following best represents the coordinates of point P such that P is $\frac{1}{3}$ of the distance from R to T where R is at $(-2, 0)$, and T is at $(4, 3)$? TEKS G.2(A) MP G.1(E)

F $(0, 1)$

G $\left(1, \frac{3}{2}\right)$

H $\left(-\frac{4}{3}, 1\right)$

J $(2, 1)$

LESSON 1-4 Angle Measure

::Then

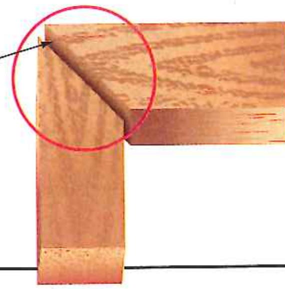
- You measured line segments.

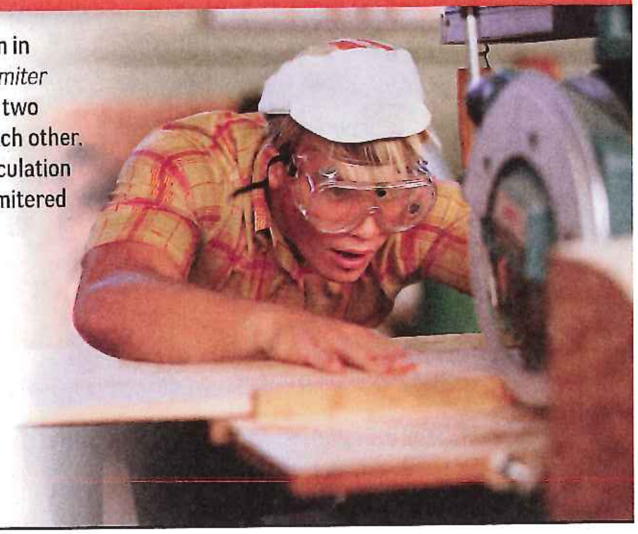
::Now

1. Measure and classify angles.
2. Identify and use congruent angles and the bisector of an angle.

::Why?

- One of the skills Dale must learn in carpentry class is how to cut a *miter* joint. This joint is created when two boards are cut at an angle to each other. He has learned that one miscalculation in angle measure can result in mitered edges that do not fit together.





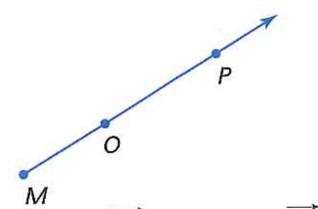
Targeted TEKS
G.5(B) Construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge.

Mathematical Processes
G.1(D) Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.
Also addresses G.1(E).

New Vocabulary
ray
opposite rays
angle
side
vertex
interior
exterior
degree
right angle
acute angle
obtuse angle
angle bisector

1 Measure and Classify Angles

A **ray** is a part of a line. It has one endpoint and extends indefinitely in one direction. Rays are named by stating the endpoint first and then any other point on the ray. The ray shown cannot be named as $\overrightarrow{OM}$ because O is not the endpoint of the ray.

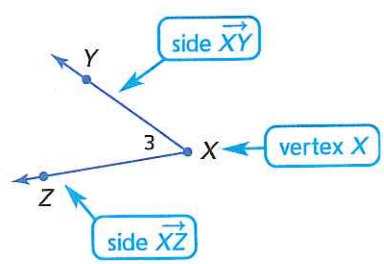


ray MP, $\overrightarrow{MP}$, ray MO, or $\overrightarrow{MO}$

If you choose a point on a line, that point determines exactly two rays called **opposite rays**. Since both rays share a common endpoint, opposite rays are collinear

$\overrightarrow{JH}$ and $\overrightarrow{JK}$ are opposite rays.

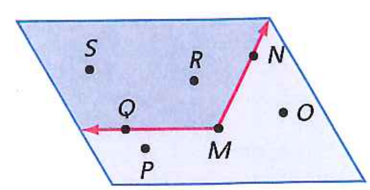
An **angle** is formed by two *noncollinear* rays that have a common endpoint. The rays are called **sides** of the angle. The common endpoint is the **vertex**.



When naming angles using three letters, the vertex must be the second of the three letters. You can name an angle using a single letter only when there is exactly one angle located at that vertex. The angle shown can be named as $\angle X$, $\angle YXZ$, $\angle ZXY$, or $\angle 3$.

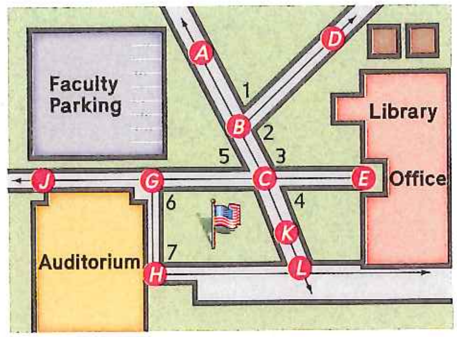
An angle divides a plane into three distinct parts.

- Points Q, M, and N lie on the angle.
- Points S and R lie in the **interior** of the angle.
- Points P and O lie in the **exterior** of the angle.



Real-World Example 1 Angles and Their Parts

MAPS **Use the map of a high school shown.**



a. Name all angles that have B as a vertex.

$\angle 1$ or $\angle ABD$, and $\angle 2$ or $\angle DBC$

b. Name the sides of $\angle 3$.

$\overrightarrow{CA}$ and $\overline{CE}$ or $\overrightarrow{CB}$ and $\overline{CE}$

c. What is another name for $\angle GHL$?

$\angle 7$, $\angle H$, or $\angle LHG$

d. Name a point in the interior of $\angle DBK$.

Point E

StudyTip

Segments as Sides Because a ray can contain a line segment, the side of an angle can be a segment.

GuidedPractice

1A. What is the vertex of $\angle 5$?

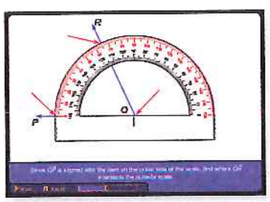
1B. Name the sides of $\angle 5$.

1C. Write another name for $\angle ECL$.

1D. Name a point in the exterior of $\angle CLH$.

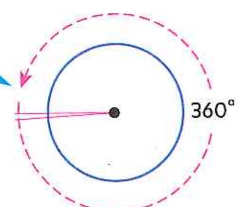
Go Online!

Look for the **Animations** icons for concepts that have video animations. Log into ConnectED to see them.



Angles are measured in units called degrees. The **degree** results from dividing the distance around a circle into 360 parts.

$1° = \frac{1}{360}$ of a turn around a circle.



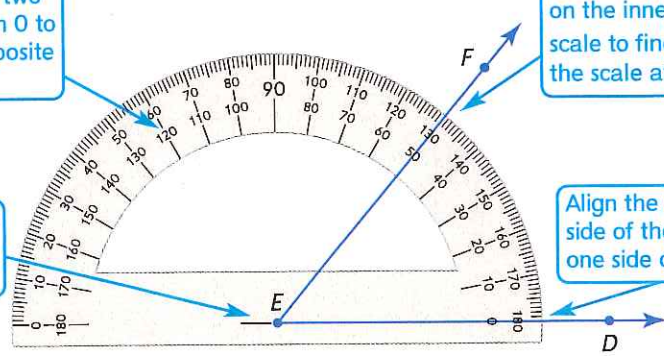
To measure an angle, you can use a *protractor*. Angle DEF below is a 50 degree (50°) angle. We say that the *degree measure* of $\angle DEF$ is 50, or $m\angle DEF = 50$.

The protractor has two scales running from 0 to 180 degrees in opposite directions.

Since $\overrightarrow{ED}$ is aligned with the 0 on the inner scale, use the inner scale to find that $\overrightarrow{EF}$ intersects the scale at 50 degrees.

Place the center point of the protractor on the vertex.

Align the 0 on either side of the scale with one side of the angle.

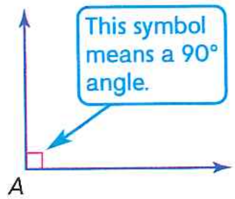
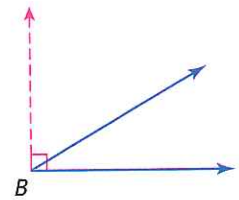
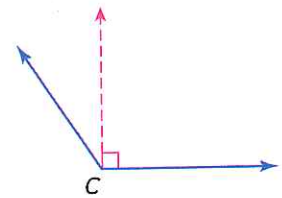


Reading Math

Straight Angle Opposite rays with the same vertex form a *straight angle*. Its measure is 180. Unless otherwise specified in this book, however, the term *angle* means a nonstraight angle.

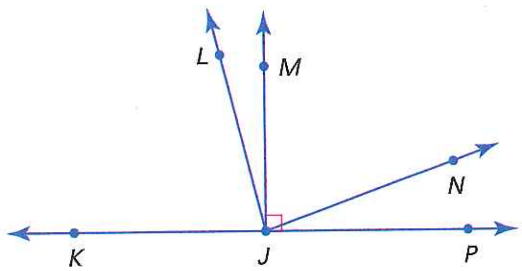
Angles can be classified by their measures as shown below.

Key Concept Classify Angles

right angle	acute angle	obtuse angle
		
$m\angle A = 90$	$m\angle B < 90$	$180 > m\angle C > 90$

Example 2 Measure and Classify Angles

Copy the diagram below, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



a. $\angle MJP$

$\angle MJP$ is marked as a right angle, so $m\angle MJP = 90$.

b. $\angle LJP$

Point L on angle $\angle LJP$ lies on the exterior of right angle $\angle MJP$, so $\angle LJP$ is an obtuse angle. Use a protractor to find that $m\angle LJP = 105$

CHECK Since $105 > 90$, $\angle LJP$ is an obtuse angle. ✓

c. $\angle NJP$

Point N on angle $\angle NJP$ lies on the interior of right angle $\angle MJP$, so $\angle NJP$ is an acute angle. Use a protractor to find that $m\angle NJP = 20$.

CHECK Since $20 < 90$, $\angle NJP$ is an acute angle. ✓

Watch Out!

Classify Before Measuring Classifying an angle before measuring it can prevent you from choosing the wrong scale on your protractor. In Example 2b, you must decide whether $\angle LJP$ measures 75 or 105. Since $\angle LJP$ is an obtuse angle, you can reason that the correct measure must be 105.

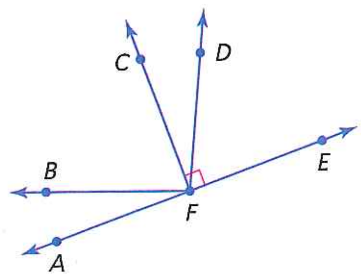
Guided Practice

2A. $\angle AFB$

2B. $\angle CFA$

2C. $\angle AFD$

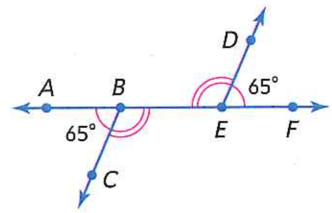
2D. $\angle CFD$



2 Congruent Angles

Just as segments that have the same measure are congruent segments, angles that have the same measure are *congruent angles*.

In the figure, since $m\angle ABC = m\angle FED$, then $\angle ABC \cong \angle FED$. Matching numbers of arcs on a figure also indicate congruent angles, so $\angle CBE \cong \angle DEB$.

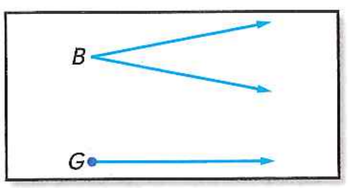


You can produce an angle congruent to a given angle using a construction.

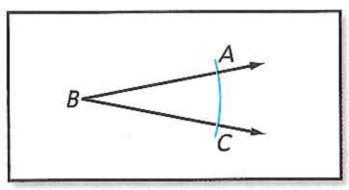
TEKS G.5(B)

Construction Copy an Angle

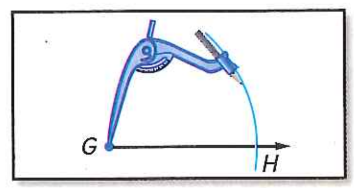
Step 1 Draw an angle like $\angle B$ on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint G.



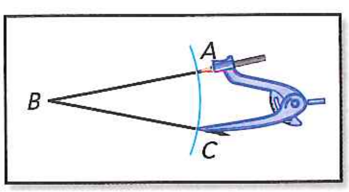
Step 2 Place the tip of the compass at point B and draw a large arc that intersects both sides of $\angle B$. Label the points of intersection A and C.



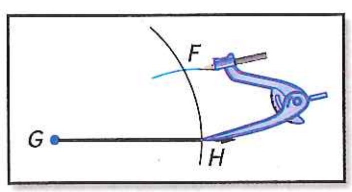
Step 3 Using the same compass setting, put the compass at point G and draw a large arc that starts above the ray and intersects the ray. Label the point of intersection H.



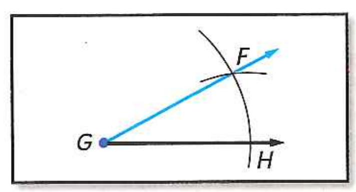
Step 4 Place the point of your compass on C and adjust so that the pencil tip is on A.



Step 5 Without changing the setting, place the compass at point H and draw an arc to intersect the larger arc you drew in Step 3. Label the point of intersection F.



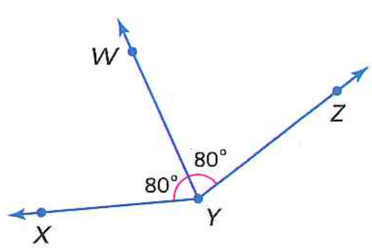
Step 6 Use a straightedge to draw $\overrightarrow{GF}$. $\angle ABC \cong \angle FGH$



StudyTip

Segments A line segment can also bisect an angle.

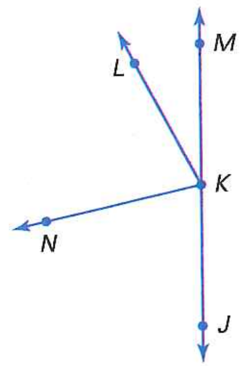
A ray that divides an angle into two congruent angles is called an **angle bisector**. If $\overrightarrow{YW}$ is the angle bisector of $\angle XYZ$, then point W lies in the interior of $\angle XYZ$ and $\angle XYW \cong \angle WYZ$.



Just as with segments, when a line, segment, or ray divides an angle into smaller angles, the sum of the measures of the smaller angles equals the measure of the largest angle. So in the figure, $m\angle XYW + m\angle WYZ = m\angle XYZ$.

Example 3 Find the Angle Measure

ALGEBRA In the figure, $\overrightarrow{KJ}$ and $\overrightarrow{KM}$ are opposite rays, and $\overrightarrow{KN}$ bisects $\angle JKL$. If $m\angle JKN = 8x - 13$ and $m\angle NKL = 6x + 11$, find $m\angle JKN$.



Step 1 Solve for x.

Since $\overrightarrow{KN}$ bisects $\angle JKL$, $\angle JKN \cong \angle NKL$.

$m\angle JKN = m\angle NKL$	Definition of congruent angles
$8x - 13 = 6x + 11$	Substitution
$8x = 6x + 24$	Add 13 to each side.
$2x = 24$	Subtract 6x from each side.
$x = 12$	Divide each side by 2.

Step 2 Use the value of x to find $m\angle JKN$.

$m\angle JKN = 8x - 13$	Given
$= 8(12) - 13$	$x = 12$
$= 96 - 13$ or 83	Simplify.

Guided Practice

3. Suppose $m\angle JKL = 9y + 15$ and $m\angle JKN = 5y + 2$. Find $m\angle JKL$.

Study Tip

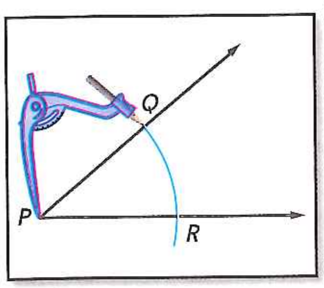
Checking Solutions Check that you have computed the value of x correctly by substituting the value into the expression for $\angle NKL$. If you don't get the same measure as $\angle JKN$, you have made an error.

You can produce the angle bisector of any angle without knowing the measure of the angle.

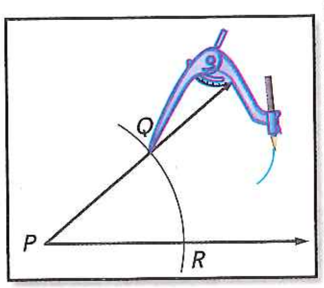
TEKS G.5(B)

Construction Bisect an Angle

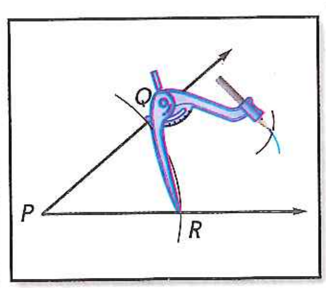
Step 1 Draw an angle. Put your compass at P and draw a large arc that intersects both sides of $\angle P$. Label the points of intersection Q and R.



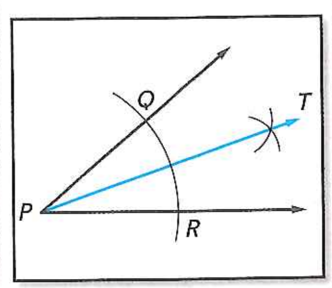
Step 2 With the compass at point Q, draw an arc in the interior of the angle.



Step 3 Keeping the same compass setting, place the compass at point R and draw an arc that intersects the arc drawn in Step 2. Label the point of intersection T.



Step 4 Draw $\overrightarrow{PT}$. $\overrightarrow{PT}$ is the bisector of $\angle P$.



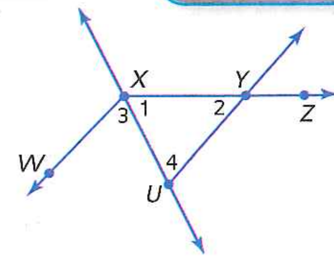
MAKE A CONJECTURE about the angles that result when you bisect an obtuse angle, a right angle, and an acute angle.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

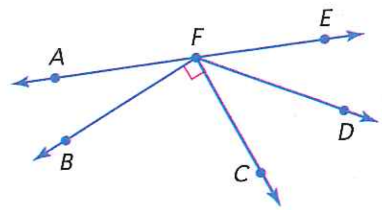
Go Online! for a Self-Check Quiz

Example 1 **Use the figure at the right.**



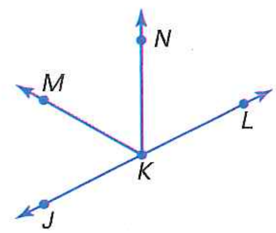
1. Name the vertex of $\angle 4$.
2. Name the sides of $\angle 3$.
3. What is another name for $\angle 2$?
4. What is another name for $\angle UXY$?

Example 2 **Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.**



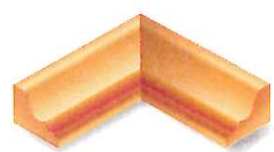
5. $\angle CFD$
6. $\angle AFD$
7. $\angle BFC$
8. $\angle AFB$

Example 3 **ALGEBRA In the figure, $\overrightarrow{KJ}$ and $\overrightarrow{KL}$ are opposite rays. $\overrightarrow{KN}$ bisects $\angle LKM$.**



9. If $m\angle LKM = 7x - 5$ and $m\angle NKM = 3x + 9$, find $m\angle LKM$.
10. If $m\angle NKL = 7x - 9$ and $m\angle JKM = x + 3$, find $m\angle JKN$.

11. **MP APPLY MATH** A miter cut is used to build picture frames with corners that meet at right angles.

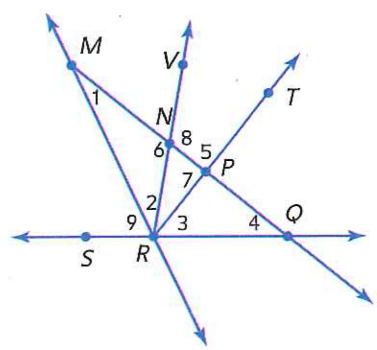


   a. José miters the ends of some wood for a picture frame at congruent angles. What is the degree measure of his cut? Explain and classify the angle.
   b. What does the joint represent in relation to the angle formed by the two pieces?

Practice and Problem Solving

Extra Practice is on page R1.

Example 1 **For Exercises 12–29, use the figure at the right.**



Name the vertex of each angle.

12. $\angle 4$
13. $\angle 7$
14. $\angle 2$
15. $\angle 1$

Name the sides of each angle.

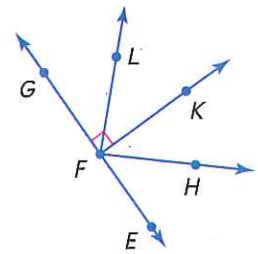
16. $\angle TPQ$
17. $\angle VNM$
18. $\angle 6$
19. $\angle 3$

Write another name for each angle.

20. $\angle 9$
21. $\angle QPT$
22. $\angle MQS$
23. $\angle 5$

24. Name an angle with vertex N that appears obtuse.
25. Name an angle with vertex Q that appears acute.
26. Name a point in the interior of $\angle VRQ$.
27. Name a point in the exterior of $\angle MRT$.
28. Name a pair of angles that share exactly one point.
29. Name a pair of angles that share more than one point.

Example 2 **Copy the diagram shown, and extend each ray. Classify each angle as *right, acute,* or *obtuse.* Then use a protractor to measure the angle to the nearest degree.**



30. $\angle GFK$

31. $\angle EFK$

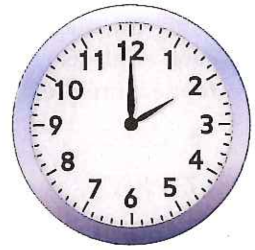
32. $\angle LFK$

33. $\angle EFH$

34. $\angle GFH$

35. $\angle EFL$

36. CLOCKS Determine at least three different times during the day when the hands on a clock form each of the following angles. Explain.

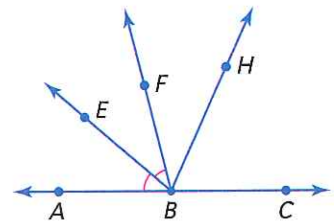


a. right angle

b. obtuse angle

c. congruent acute angles

Example 3 **ALGEBRA In the figure, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. $\overrightarrow{BH}$ bisects $\angle EBC$.**



37 If $m\angle ABE = 2n + 7$ and $m\angle EBF = 4n - 13$, find $m\angle ABE$.

38. If $m\angle EBH = 6x + 12$ and $m\angle HBC = 8x - 10$, find $m\angle EBH$.

39. If $m\angle ABF = 7b - 24$ and $m\angle ABE = 2b$, find $m\angle EBF$.

40. If $m\angle EBC = 31a - 2$ and $m\angle EBH = 4a + 45$, find $m\angle HBC$.

41. If $m\angle ABF = 8s - 6$ and $m\angle ABE = 2(s + 11)$, find $m\angle EBF$.

42. If $m\angle EBC = 3r + 10$ and $m\angle ABE = 2r - 20$, find $m\angle EBF$.

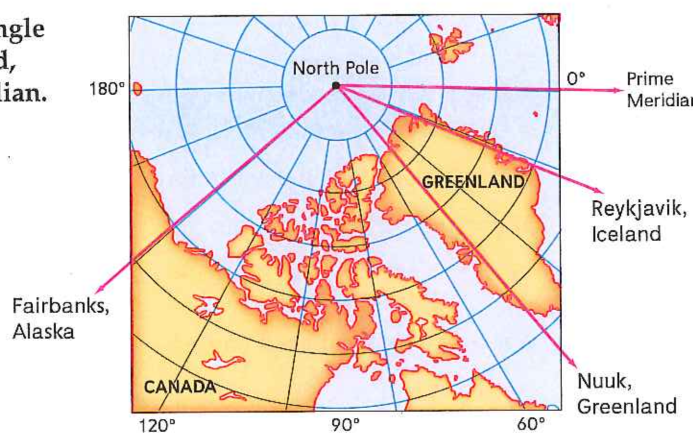
43. MAPS Estimate the measure of the angle formed by each city or location listed, the North Pole, and the Prime Meridian.

a. Nuuk, Greenland

b. Fairbanks, Alaska

c. Reykjavik, Iceland

d. Prime Meridian



44. MP TOOLS AND TECHNIQUES A compass rose is a design on a map that shows directions. In addition to the directions of north, south, east, and west, a compass rose can have as many as 32 markings.

a. With the center of the compass as its vertex, what is the measure of the angle between due west and due north?

b. What is the measure of the angle between due north and north-west?

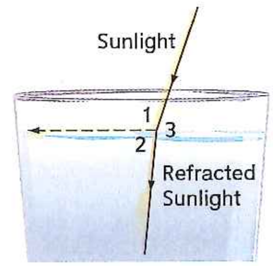
c. How does the north-west ray relate to the angle in part **a**?

Plot the points in a coordinate plane and sketch ∠XYZ. Then classify it as *right, acute,* or *obtuse.*

45. $X(5, -3), Y(4, -1), Z(6, -2)$

46. $X(6, 7), Y(2, 3), Z(4, 1)$

47 **PHYSICS** When you look at a pencil in water, it looks bent. This illusion is due to *refraction*, or the bending of light when it moves from one substance to the next.

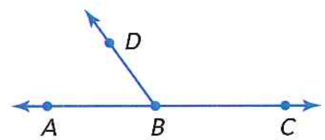


a. What is $m\angle 1$? Classify this angle as *acute, right,* or *obtuse.*

b. What is $m\angle 2$? Classify this angle as *acute, right,* or *obtuse.*

c. Without measuring, determine how many degrees the path of the light changes after it enters the water. Explain your reasoning.

48. MP **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship of angles that compose opposite rays.



a. Geometric Copy the figure shown. Draw three additional figures, varying the placement of point *D*. Use a protractor to measure ∠*ABD* and ∠*DBC* for each figure.

b. Tabular Organize the measures for each figure into a table. Include a column in your table to record the sum of these measures.

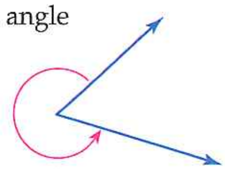
c. Verbal Make a conjecture about the sum of the measures of the two angles. Explain your reasoning.

d. Algebraic If *x* is the measure of ∠*ABD* and *y* is the measure of ∠*DBC*, write an equation that relates the two angle measures.

H.O.T. Problems Use Higher-Order Thinking Skills

49. MP **ORGANIZE IDEAS** Draw an obtuse angle named *ABC*. Measure ∠*ABC*. Construct an angle bisector $\overrightarrow{BD}$ of ∠*ABC*. Explain the steps in your construction and justify each step. Classify the two angles formed by the angle bisector.

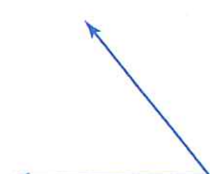
50. MP **TOOLS AND TECHNIQUES** Use a compass and a straightedge to construct an angle congruent to the angle shown. How would you use a protractor to find the measure of the angle?



51. MP **JUSTIFY ARGUMENTS** Is the sum of two acute angles *sometimes, always,* or *never* an obtuse angle? Explain.

52. MP **ANALYZE RELATIONSHIPS** $\overrightarrow{MP}$ bisects ∠*LMN*, $\overrightarrow{MQ}$ bisects ∠*LMP*, and $\overrightarrow{MR}$ bisects ∠*QMP*. If $m\angle RMP = 21$, find $m\angle LMN$. Explain your reasoning.

53. **WRITING IN MATH** Rashid says that he can estimate the measure of an acute angle using a piece of paper to within six degrees of accuracy. Explain how this would be possible. Then use this method to estimate the measure of the angle shown.

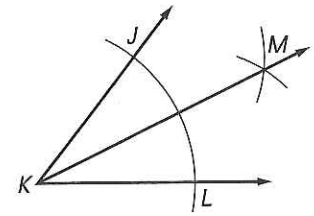


Preparing for Assessment

Example

TEKS G.5(B) MP G.1(D), G.1(F)

TEKS REVIEW Mei used a compass and straightedge to make the construction shown here.



Which of the following statements must be true?

A $m\angle JKM = \frac{1}{2}m\angle JKL$

B $m\angle JKL = \frac{1}{2}m\angle MKL$

C $m\angle JKM = m\angle JKL$

D $m\angle MKL = 2m\angle JKL$

Mei constructed the angle bisector of $\angle JKL$. This means $\angle MKL \cong \angle JKM$.

$m\angle JKM + m\angle MKL = m\angle JKL$

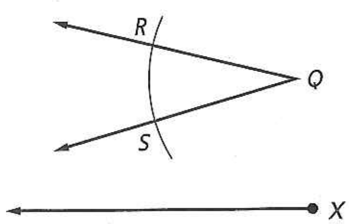
$m\angle JKM + m\angle JKM = m\angle JKL$ — Since $\angle MKL \cong \angle JKM$, $m\angle MKL = m\angle JKM$.

$2m\angle JKM = m\angle JKL$ — Add.

$m\angle JKM = \frac{1}{2}m\angle JKL$ — Divide each side by 2.

The correct answer is choice A.

54. Tyrell is constructing an angle congruent to $\angle Q$. The work he has done so far is shown below.



Which of the following best describes what Tyrell should do next? TEKS G.5(B) MP G.1(D), G.1(F)

A Adjust the compass setting to measure the distance from point R to point S.

B Place the point of the compass on point R.

C Place the point of the compass on point S.

D Place the point of the compass on point X.

55. Which construction requires you to draw only one arc? TEKS G.5(B) MP G.1(D), G.1(F)

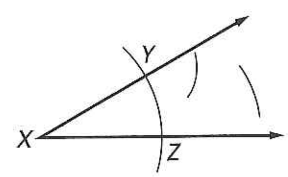
F Copy a segment

G Bisect a segment

H Copy an angle

J Bisect an angle

56. **ACT/SAT** A student was asked to construct the angle bisector of $\angle X$. The figure shows the work the student has done so far.



Which of the following best describes the error the student made?

A The arcs are not long enough.

B The student used the steps for constructing an angle that is congruent to $\angle X$.

C The student changed the compass setting when drawing the arcs.

D The student should not have drawn the arc that passes through points Y and Z.

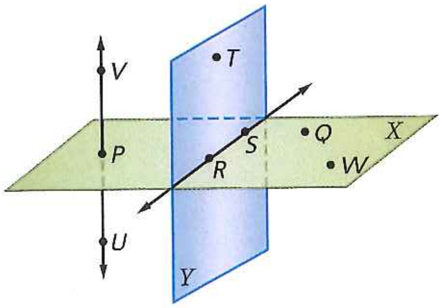
E The student should have used a protractor.

57. **GRIDDABLE** Naomi constructed a ray, $\overrightarrow{BD}$, that is the bisector of $\angle ABC$. Given that $m\angle ABD = 65.2$, what is $m\angle ABC$? TEKS G.5(B) MP G.1(C), G.1(F)

CHAPTER 1 Mid-Chapter Quiz

Lessons 1-1 through 1-4

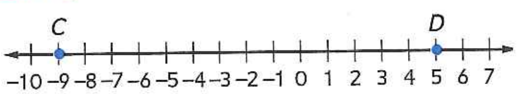
Use the figure to complete each of the following. (Lesson 1-1)



1. Name another point that is collinear with points *U* and *V*.
2. What is another name for plane *Y*?
3. Name a line that is coplanar with points *P*, *Q*, and *W*.

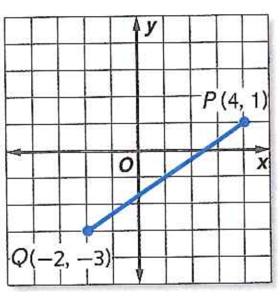
Find the value of x and AC if B is between points A and C. (Lesson 1-2)

4. $AB = 12, BC = 8x - 2, AC = 10x$
5. $AB = 5x, BC = 9x - 2, AC = 11x + 7.6$
6. Find CD and the coordinate of the midpoint of $\overline{CD}$. (Lesson 1-3)

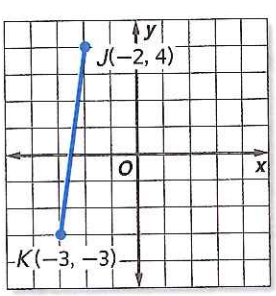


Find the coordinates of the midpoint of each segment. Then find the length of each segment. (Lesson 1-3)

7.



8.

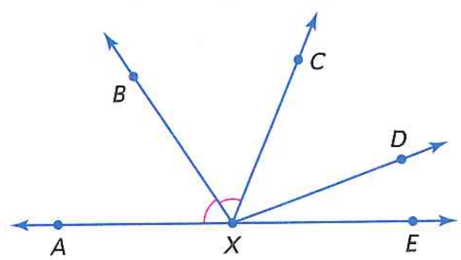


Find the coordinates of the midpoint of a segment with the given endpoints. Then find the distance between each pair of points. (Lesson 1-3)

9. $P(26, 12)$ and $Q(8, 42)$
10. $M(6, -41)$ and $N(-18, -27)$
11. **MAPS** A map of a town is drawn on a coordinate grid. The high school is found at point (3, 1) and town hall is found at (−5, 7). (Lesson 1-3)
    a. If the high school is at the midpoint between the town hall and the town library, at which ordered pair should you find the library?
    b. If one unit on the grid is equivalent to 50 meters, how far is the high school from town hall?
12. **MULTIPLE CHOICE** The vertex of $\angle ABC$ is located at the origin. Point A is located at (5, 0) and Point C is located at (0, 2). How can $\angle ABC$ be classified? (Lesson 1-4)

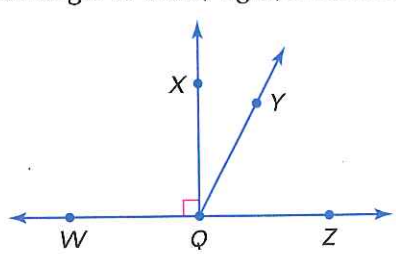
    A acute **C** right
    B obtuse **D** scalene

In the figure, $\overrightarrow{XA}$ and $\overrightarrow{XE}$ are opposite rays, and $\angle AXC$ is bisected by $\overrightarrow{XB}$. (Lesson 1-4)



13. If $m\angle AXC = 8x - 7$ and $m\angle AXB = 3x + 10$, find $m\angle AXC$.
14. If $m\angle CXD = 4x + 6$, $m\angle DXE = 3x + 1$, and $m\angle CXE = 8x - 2$, find $m\angle DXE$.

Classify each angle as *acute*, *right*, or *obtuse*. (Lesson 1-4)



15. $\angle WQY$ 16. $\angle YQZ$