

# Expressions, Equations, and Functions



## Targeted TEKS

A.10(D), A.5(A), A.2(A), A.12(A),  
A.12(B), A.3(C), A.7(A)



## Mathematical Processes

A.1(A), A.1(B), A.1(C), A.1(D), A.1(E),  
A.1(F), A.1(G)

## THEN

You have learned how to perform operations on whole numbers.

## NOW

In this chapter, you will:

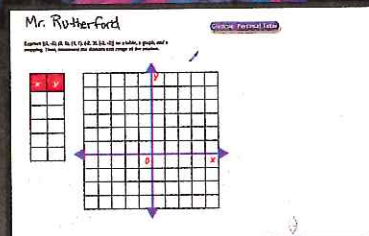
- Write algebraic expressions.
- Use the order of operations.
- Solve equations.
- Represent and interpret relations and functions.
- Use function notation.
- Interpret the graphs of functions.

## WHY

**SCUBA DIVING** A scuba diving store near Balmorhea State Park rents air tanks and wet suits. An algebraic expression can be written to represent the total cost to rent this equipment. This expression can be evaluated to determine the cost for a group of people to rent the equipment.

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ALEKS



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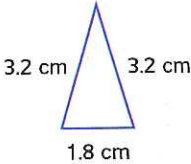
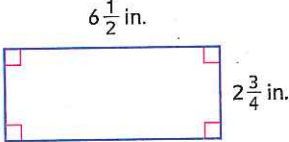
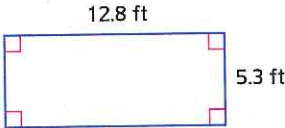
# Get Ready for the Chapter



**Go Online!** Take the Chapter Readiness Quiz online as another option

**Diagnose Readiness** | You have two options for checking prerequisite skills.

**Textbook Option** Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck	QuickReview
<p>Write each fraction in simplest form. If the fraction is already in simplest form, write <i>simplest form</i>.</p> <p>1. <math>\frac{24}{36}</math>      2. <math>\frac{34}{85}</math>      3. <math>\frac{36}{12}</math></p> <p>4. <math>\frac{27}{45}</math>      5. <math>\frac{11}{18}</math>      6. <math>\frac{5}{65}</math></p> <p>7. <math>\frac{19}{1}</math>      8. <math>\frac{16}{44}</math>      9. <math>\frac{64}{88}</math></p> <p>10. <b>ICE CREAM</b> Fifty-four out of 180 customers said that cookie dough ice cream was their favorite flavor. What fraction of customers was this?</p>	<p><b>Example 1</b></p> <p>Write <math>\frac{24}{40}</math> in simplest form.</p> <p>Find the greatest common factor (GCF) of 24 and 40.</p> <p>factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 factors of 40: 1, 2, 4, 5, 8, 10, 20, 40</p> <p>The GCF of 24 and 40 is 8.</p> <p><math>\frac{24 \div 8}{40 \div 8} = \frac{3}{5}</math>      Divide the numerator and denominator by their GCF, 8.</p>
<p>Find the perimeter of each figure.</p> <p>11. </p> <p>12. </p> <p>13. <b>FENCING</b> Jolon needs to fence a rectangular garden. The dimensions of the garden are 6 meters by 4 meters. How much fencing does Jolon need to purchase?</p>	<p><b>Example 2</b></p> <p>Find the perimeter.</p> <p></p> <p><math>P = 2\ell + 2w</math></p> <p><math>= 2(12.8) + 2(5.3)</math>      <math>\ell = 12.8</math> and <math>w = 5.3</math></p> <p><math>= 25.6 + 10.6</math> or 36.2      Simplify.</p> <p>The perimeter is 36.2 feet.</p>
<p>Evaluate.</p> <p>14. <math>6 \cdot \frac{2}{3}</math>      15. <math>4.2 \cdot 8.1</math>      16. <math>\frac{3}{8} \div \frac{1}{4}</math></p> <p>17. <math>5.13 \div 2.7</math>      18. <math>3\frac{1}{5} \cdot \frac{3}{4}</math>      19. <math>2.8 \cdot 0.2</math></p> <p>20. <b>CONSTRUCTION</b> A board measuring 72 feet must be cut into three equal pieces. Find the length of each piece.</p>	<p><b>Example 3</b></p> <p>Find <math>2\frac{1}{4} \div 1\frac{1}{2}</math>.</p> <p><math>2\frac{1}{4} \div 1\frac{1}{2} = \frac{9}{4} \div \frac{3}{2}</math>      Write mixed numbers as improper fractions.</p> <p><math>= \frac{9}{4} \left(\frac{2}{3}\right)</math>      Multiply by the reciprocal.</p> <p><math>= \frac{18}{12}</math> or <math>1\frac{1}{2}</math>      Simplify.</p>

# Get Started on the Chapter



**Go Online!** for Vocabulary Review Games and key vocabulary in 13 languages

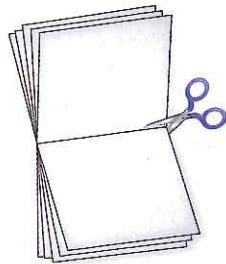
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 1. To get ready, identify important terms and organize your resources. Working with a partner can be helpful as you prepare and as you read the chapter.

## FOLDABLES®

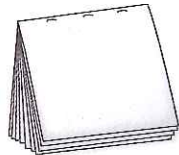
### Study Organizer

**Expressions, Equations, and Functions** Make this Foldable to help you organize your Chapter 1 notes about expressions, equations, and functions. Begin with five sheets of plain paper.

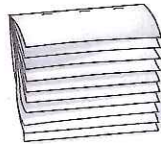
- 1** Fold the sheets of paper in half along the width. Then cut along the crease.



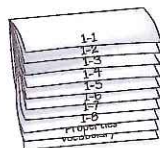
- 2** Staple the ten half-sheets together to form a booklet.



- 3** Cut nine centimeters from the bottom of the top sheet, eight centimeters from the second sheet, and so on.



- 4** Label each of the tabs with a lesson number. The ninth tab is for Properties and the last tab is for Vocabulary.



### New Vocabulary

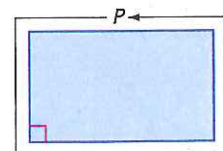
English		Español
algebraic expression	p. 5	expression algebraica
variable	p. 5	variable
term	p. 5	término
power	p. 5	potencia
coefficient	p. 28	coeficiente
equation	p. 33	ecuación
solution	p. 33	solución
identity	p. 35	identidad
relation	p. 40	relación
domain	p. 40	domino
range	p. 40	rango
independent variable	p. 42	variable independiente
dependent variable	p. 42	variable dependiente
function	p. 47	función
intercept	p. 56	intersección
line symmetry	p. 57	simetría
end behavior	p. 57	comportamiento final

### Review Vocabulary

**additive inverse** **inverso aditivo** a number and its opposite

**multiplicative inverse** **inverso multiplicativo** two numbers with a product of 1

**perimeter** **perímetro** the distance around a geometric figure





### Then

- You performed operations on integers.

### Now

- Write verbal expressions for algebraic expressions.
- Write algebraic expressions for verbal expressions.

### Why?

- Cassie and her friends are at a baseball game. Rangers Ballpark is running a promotion where hot dogs are \$0.10 each. Suppose  $d$  represents the number of hot dogs Cassie and her friends eat. Then  $0.10d$  represents the cost of the hot dogs they eat.



#### Targeted TEKS

**Preparation for A.2(C)**  
Write linear equations in two variables given a table of values, a graph, and a verbal description.



#### Mathematical Processes

**A.1(E)** Create and use representations to organize, record, and communicate mathematical ideas.



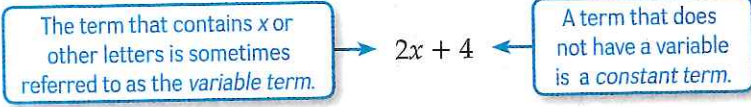
#### New Vocabulary

- algebraic expression
- variable
- term
- factor
- product
- power
- exponent
- base

**1 Write Verbal Expressions** An **algebraic expression** consists of sums and/or products of numbers and variables. In the algebraic expression  $0.10d$ , the letter  $d$  is called a variable. In algebra, **variables** are symbols used to represent unspecified numbers or values. Any letter may be used as a variable.

$$0.10d \quad 2x + 4 \quad 3 + \frac{z}{6} \quad p \cdot q \quad 4cd \div 3mn$$

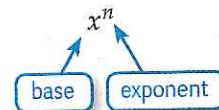
A **term** of an expression may be a number, a variable, or a product or quotient of numbers and variables. For example,  $0.10d$ ,  $2x$  and  $4$  are each terms.



In a multiplication expression, the quantities being multiplied are **factors**, and the result is the **product**. A raised dot or set of parentheses are often used to indicate a product. Here are several ways to represent the product of  $x$  and  $y$ .

$$xy \quad x \cdot y \quad x(y) \quad (x)y \quad (x)(y)$$

An expression like  $x^n$  is called a **power**. The word *power* can also refer to the exponent. The **exponent** indicates the number of times the base is used as a factor. In an expression of the form  $x^n$ , the **base** is  $x$ . The expression  $x^n$  is read “ $x$  to the  $n$ th power.” When no exponent is shown, it is understood to be 1. For example,  $a = a^1$ .



#### Example 1 Write Verbal Expressions

Write a verbal expression for each algebraic expression.

- $3x^4$   
three times  $x$  to the fourth power
- $5z^2 + 16$   
5 times  $z$  to the second power plus sixteen

#### Guided Practice

- $16u^2 - 3$
- $\frac{1}{2}a + \frac{6b}{7}$

#### StudyTip

##### Using Your Text

Notice that new terms are listed at the beginning of the lesson and also highlighted in context.

## 2 Write Algebraic Expressions

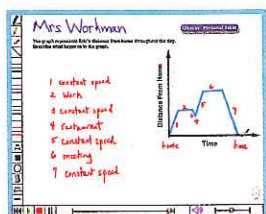
Another important skill is translating verbal expressions into algebraic expressions.

### Key Concept Translating Verbal to Algebraic Expressions

Operation	Verbal Phrases
Addition	more than, sum, plus, increased by, added to
Subtraction	less than, subtracted from, difference, decreased by, minus
Multiplication	product of, multiplied by, times, of
Division	quotient of, divided by

### Go Online!

**Personal Tutors** for each example let you follow along as a teacher solves a problem. Pause and rewind as you need.



### Example 2 Write Algebraic Expressions

Write an algebraic expression for each verbal expression.

a. a number  $t$  more than 6

The words *more than* suggest addition.  
Thus, the algebraic expression is  $6 + t$  or  $t + 6$ .

b. 10 less than the product of 7 and  $f$

*Less than* implies subtraction, and *product* suggests multiplication.  
So the expression is written as  $7f - 10$ .

c. two thirds of the volume  $v$

The word *of* with a fraction implies that you should multiply.  
The expression could be written as  $\frac{2}{3}v$  or  $\frac{2v}{3}$ .

### Guided Practice

2A. the product of  $p$  and 6

2B. one third of the area  $a$

Variables can represent quantities that are known and quantities that are unknown. They are also used in formulas, expressions, and equations.

### Real-World Example 3 Write an Expression

**SPORTS MARKETING** Mr. Martinez orders 250 key chains printed with his athletic team's logo and 500 pencils printed with their Web address. Write an algebraic expression that represents the cost of the order.

Let  $k$  be the cost of each key chain and  $p$  be the cost of each pencil. Then the cost of the key chains is  $250k$  and the cost of the pencils is  $500p$ . The cost of the order is represented by  $250k + 500p$ .

### Guided Practice

3. **COFFEE SHOP** Katie bakes 40 pastries and makes coffee for 200 people. Write an algebraic expression to represent the cost of this situation.

### Real-World Career

#### Sports Marketing

Sports marketers promote and manage athletes, teams, facilities and sports-related businesses and organizations. A minimum of a bachelor's degree in sports management or business administration is preferred.





**Example 1** Write a verbal expression for each algebraic expression.

1.  $2m$                                       2.  $\frac{2}{3}r^4$                                       3.  $a^2 - 18b$

**Example 2** Write an algebraic expression for each verbal expression.

4. the sum of a number and 14                                      5. 6 less a number  $t$   
 6. 7 more than 11 times a number                                      7. 1 minus the quotient of  $r$  and 7  
 8. two fifths of the square of a number  $j$                                       9.  $n$  cubed increased by 5

**Example 3** 10. **GROCERIES** Mr. Bailey purchased some groceries that cost  $d$  dollars. He paid with a \$50 bill. Write an expression for the amount of change he will receive.

**Practice and Problem Solving**

Extra Practice is on page R1.

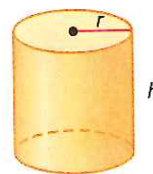
**Example 1** Write a verbal expression for each algebraic expression.

11.  $4q$                                       12.  $\frac{1}{8}y$                                       13.  $15 + r$                                       14.  $w - 24$   
 15.  $3x^2$                                       16.  $\frac{r^4}{9}$                                       17.  $2a + 6$                                       18.  $r^4 \cdot t^3$

**Example 2** Write an algebraic expression for each verbal expression.

19.  $x$  more than 7                                      20. a number less 35  
 21. 5 times a number                                      22. one third of a number  
 23.  $f$  divided by 10                                      24. the quotient of 45 and  $r$   
 25. three times a number plus 16                                      26. 18 decreased by 3 times  $d$   
 27.  $k$  squared minus 11                                      28. 20 divided by  $t$  to the fifth power

**Example 3** 29. **GEOMETRY** The volume of a cylinder is  $\pi$  times the radius  $r$  squared multiplied by the height  $h$ . Write an expression for the volume.



30. **FINANCIAL LITERACY** Jocelyn makes  $x$  dollars per hour working at the grocery store and  $n$  dollars per hour babysitting. Write an expression that describes her earnings if she babysat for 25 hours and worked at the grocery store for 15 hours.

Write a verbal expression for each algebraic expression.

31.  $25 + 6x^2$                                       32.  $6f^2 + 5f$                                       33.  $\frac{3a^5}{2}$

34. **MP ORGANIZE IDEAS** A local gym membership costs \$20 per month plus additional activity charges. If  $x$  is the number of exercise classes taken above the planned amount and  $y$  is the number of massages taken above the planned amount, interpret each expression.

- a.  $15x$   
 b.  $55y$   
 c.  $15x + 55y + 20$

- 35. DREAMS** It is believed that about  $\frac{3}{4}$  of our dreams involve people that we know.
- Write an expression to describe the number of dreams that feature people you know if you have  $d$  dreams.
  - Use the expression you wrote to predict the number of dreams that include people you know out of 28 dreams.
- 36. SPORTS** In football, a touchdown is awarded 6 points and the team then may attempt a kick for a point after a touchdown.
- Write an expression that describes the number of points scored on touchdowns  $T$  and points after touchdowns  $p$  by one team in a game.
  - If a team wins a football game 27-0, write an equation to represent the possible number of touchdowns and points after touchdowns by the winning team.
  - If a team wins a football game 21-7, how many possible number of touchdowns and points after touchdowns were scored during the game by both teams?
- 37. MP MULTIPLE REPRESENTATIONS** In this problem, you will explore the multiplication of powers with like bases.

- a. **Tabular** Copy and complete the table.

$10^2$	$\times$	$10^1$	$=$	$10 \times 10 \times 10$	$=$	$10^3$
$10^2$	$\times$	$10^2$	$=$	$10 \times 10 \times 10 \times 10$	$=$	$10^4$
$10^2$	$\times$	$10^3$	$=$	$10 \times 10 \times 10 \times 10 \times 10$	$=$	?
$10^2$	$\times$	$10^4$	$=$	?	$=$	?

- b. **Algebraic** Write an equation for the pattern in the table.
- c. **Verbal** Make a conjecture about the exponent of the product of two powers with like bases.

### H.O.T. Problems Use Higher-Order Thinking Skills

- 38. MP ANALYZE RELATIONSHIPS** Explain the differences between an algebraic expression and a verbal expression.
- 39. MP ORGANIZE IDEAS** Define a variable to represent a real-life quantity, such as time in minutes or distance in feet. Then use the variable to write an algebraic expression to represent one of your daily activities. Describe in words what your expression represents, and explain your reasoning.
- 40. ERROR ANALYSIS** Consuelo and James are writing an algebraic expression for *three times the sum of  $n$  squared and 3*. Is either of them correct? Explain your reasoning.

Consuelo

$$3(n^2 + 3)$$

James

$$3n^2 + 3$$

- 41. MP ORGANIZE IDEAS** For the cube,  $x$  represents a positive whole number. Find the value of  $x$  such that the volume of the cube and 6 times the area of one of its faces have the same value.
- 42. WRITING IN MATH** Describe how to write an algebraic expression from a real-world situation. Include a definition of algebraic expression in your own words.



## Example

TEKS A.2(C) MP A.1(D), A.1(E)

Which equation best represents the verbal description “the quotient of  $a$  and 2 is 16”?

- A  $\frac{a}{2} = 16$
- B  $\frac{a}{2} - 16$
- C  $a + 2 = 16$
- D  $\frac{a + 2}{16}$

Read the verbal description. The quotient of  $a$  and 2 is  $a$  divided by 2, also written as  $a \div 2$  or  $\frac{a}{2}$ . The word *is* can be represented using an equals sign (=).

$\frac{a}{2}$  Write an algebraic expression for the quotient of  $a$  and 2.

$\frac{a}{2} = 16$  Set the expression equal to 16.

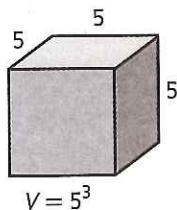
The equation is  $\frac{a}{2} = 16$ . The correct answer is choice A.

43. What is an equation for “five more than the product of 7 and a number  $t$  is 10?” TEKS A.2(C)

MP A.1(D), A.1(E)

- A  $5 > 7t + 10$
- B  $7t + 5 = 10$
- C  $5t + 7 = 10$
- D  $5 \cdot 7t + 10$

44. The volume of this cube can be expressed as  $5^3$ .



Which equation can be used to find the volume of a cube with edges that are  $x$  units shorter? TEKS A.2(C) MP A.1(A), A.1(F)

- F  $V = (x - 5)^3$
- G  $V = 5^3 - x^3$
- H  $V = (5 - x)^3$
- J  $5 - x$

45. Elsie buys a pizza for \$16 and several bottles of water for \$2 each. Let  $C$  represent the total amount of money that Elsie spends and let  $w$  represent how many bottles of water she buys. Which equation best represents this situation?

TEKS A.2(C) MP A.1(A), A.1(F)

- A  $C = 2 + 16w$
- B  $C = 16 + 2 + w$
- C  $C = 16 + 2w$
- D  $C = 2(w + 16)$

46. Which equation best describes the data in the table? TEKS A.2(C) MP A.1(E), A.1(F)

$x$	8	4	2
$y$	2	-2	-4

- F  $y = x \div 4$
- G  $y = -0.5x$
- H  $y = x + 6$
- J  $y = x - 6$



### Then

- You expressed algebraic expressions verbally.

### Now

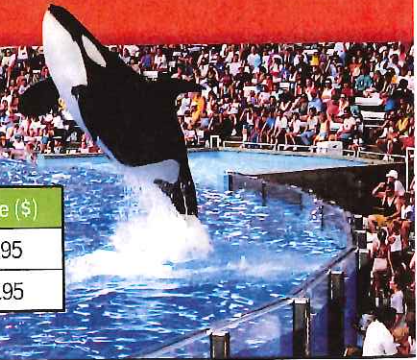
- Evaluate numerical expressions by using the order of operations.
- Evaluate algebraic expressions by using the order of operations.

### Why?

- The admission prices for SeaWorld Theme Park in San Antonio, Texas, are shown in the table. If four adults and three children go to the park, the expression below represents the cost of admission for the group.

$$4(78.95) + 3(68.95)$$

Ticket	Price (\$)
Adult	78.95
Child	68.95



#### Targeted TEKS

**Preparation for A.5(A)**  
Solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.



#### Mathematical Processes

**A.1(A)** Apply mathematics to problems arising in everyday life, society, and the workplace.

**A.1(E)** Create and use representations to organize, record, and communicate mathematical ideas.



#### New Vocabulary

evaluate  
order of operations

**1 Evaluate Numerical Expressions** To find the cost of admission, the expression  $4(78.95) + 3(68.95)$  must be evaluated. To **evaluate** an expression means to find its value.

#### Example 1 Evaluate Expressions

Evaluate  $3^5$ .

$$\begin{aligned} 3^5 &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 && \text{Use 3 as a factor 5 times.} \\ &= 243 && \text{Multiply.} \end{aligned}$$

#### Guided Practice

1A.  $2^4$

1B.  $4^5$

1C.  $7^3$

The numerical expression that represents the cost of admission contains more than one operation. The rule that lets you know which operation to perform first is called the **order of operations**.

#### Key Concept Order of Operations

**Step 1** Evaluate expressions inside grouping symbols.

**Step 2** Evaluate all powers.

**Step 3** Multiply and/or divide from left to right.

**Step 4** Add and/or subtract from left to right.

#### Example 2 Order of Operations

Evaluate  $16 - 8 \div 2^2 + 14$ .

$$\begin{aligned} 16 - 8 \div 2^2 + 14 &= 16 - 8 \div 4 + 14 && \text{Evaluate powers.} \\ &= 16 - 2 + 14 && \text{Divide 8 by 4.} \\ &= 14 + 14 && \text{Subtract 2 from 16.} \\ &= 28 && \text{Add 14 and 14.} \end{aligned}$$

#### Guided Practice

2A.  $3 + 42 \cdot 2 - 5$

2B.  $20 - 7 + 8^2 - 7 \cdot 11$

### StudyTip

**Grouping Symbols** Grouping symbols such as parentheses ( ), brackets [ ], and braces { } are used to clarify or change the order of operations. A fraction bar is considered a grouping symbol. So, evaluate expressions in the numerator and denominator before completing the division.

When one or more grouping symbols are used, evaluate within the innermost grouping symbols first.

### Example 3 Expressions with Grouping Symbols

Evaluate each expression.

a.  $4 \div 2 + 5(10 - 6)$

$$\begin{aligned} 4 \div 2 + 5(10 - 6) &= 4 \div 2 + 5(4) \\ &= 2 + 5(4) \\ &= 2 + 20 \\ &= 22 \end{aligned}$$

Evaluate inside parentheses.

Divide 4 by 2.

Multiply 5 by 4.

Add 2 to 20.

b.  $6[32 - (2 + 3)^2]$

$$\begin{aligned} 6[32 - (2 + 3)^2] &= 6[32 - (5)^2] \\ &= 6[32 - 25] \\ &= 6[7] \\ &= 42 \end{aligned}$$

Evaluate innermost expression first.

Evaluate power.

Subtract 25 from 32.

Multiply.

c.  $\frac{2^3 - 5}{15 + 9}$

$$\begin{aligned} \frac{2^3 - 5}{15 + 9} &= \frac{8 - 5}{15 + 9} \\ &= \frac{3}{15 + 9} \\ &= \frac{3}{24} \text{ or } \frac{1}{8} \end{aligned}$$

Evaluate the power in the numerator.

Subtract 5 from 8 in the numerator.

Add 15 and 9 in denominator, and simplify.

### Guided Practice

3A.  $5 \cdot 4(10 - 8) + 20$

3B.  $15 - [10 + (3 - 2)^2] + 6$

3C.  $\frac{(4 + 5)^2}{3(7 - 4)}$

**2 Evaluate Algebraic Expressions** To evaluate an algebraic expression, replace the variables with their values. Then find the value of the numerical expression using the order of operations.

### Example 4 Evaluate an Algebraic Expression

Evaluate  $3x^2 + (2y + z^3)$  if  $x = 4$ ,  $y = 5$ ,  $z = 3$ .

$$\begin{aligned} 3x^2 + (2y + z^3) &= 3(4)^2 + (2 \cdot 5 + 3^3) \\ &= 3(4)^2 + (2 \cdot 5 + 27) \\ &= 3(4)^2 + (10 + 27) \\ &= 3(4)^2 + (37) \\ &= 3(16) + 37 \\ &= 48 + 37 \\ &= 85 \end{aligned}$$

Replace  $x$  with 4,  $y$  with 5, and  $z$  with 3.

Evaluate  $3^3$ .

Multiply 2 by 5.

Add 10 to 27.

Evaluate  $4^2$ .

Multiply 3 by 16.

Add 48 to 37.

### Guided Practice

Evaluate each expression.

4A.  $a^2(3b + 5) \div c$  if  $a = 2$ ,  $b = 6$ ,  $c = 4$

4B.  $5d + (6f - g)$  if  $d = 4$ ,  $f = 3$ ,  $g = 12$

### Go Online!



Graphing calculators and scientific calculators will use the order of operations when you enter the grouping symbols in an expression.





### Real-WorldLink

The National Oceanic & Atmospheric Administration (NOAA) developed the Science on a Sphere system to educate people about Earth's processes.

Source: NOAA

## Real-World Example 5 Write and Evaluate an Expression

**ENVIRONMENTAL STUDIES** Science on a Sphere (SOS)<sup>®</sup> demonstrates the effects of atmospheric storms, climate changes, and ocean temperature on the environment. The volume of a sphere is four thirds of  $\pi$  multiplied by the radius  $r$  to the third power.

a. Write an expression that represents the volume of a sphere.

Words	four thirds	of	$\pi$ multiplied by radius to the third power
Variable	Let $r =$ radius.		
Equation	$\frac{4}{3}$	$\times$	$\pi r^3$ or $\frac{4}{3}\pi r^3$

b. Find the volume of the 3-foot radius sphere used for SOS.

$$\begin{aligned}
 V &= \frac{4}{3}\pi r^3 && \text{Volume of a sphere} \\
 &= \frac{4}{3}\pi(3)^3 && \text{Replace } r \text{ with } 3. \\
 &= \left(\frac{4}{3}\right)\pi(27) && \text{Evaluate } 3^3 = 27. \\
 &= 36\pi && \text{Multiply } \frac{4}{3} \text{ by } 27.
 \end{aligned}$$

The volume of the sphere is  $36\pi$  cubic feet.

### Guided Practice

5. **FOREST FIRES** According to the Texas Fire Incident Reporting System, an average of 1047 fires each year are started by open flame and cooking is responsible for an average of 4364 fires each year.

- Write an algebraic expression for the total number of fires in  $f$  years.
- How many total fires would there be over a 5-year period?

## Check Your Understanding

= Step-by-Step Solutions begin on page R13.



Go Online! for a Self-Check Quiz

**Examples 1-3** Evaluate each expression.

- $9^2$
- $4^4$
- $3^5$
- $30 - 14 \div 2$
- $5 \cdot 5 - 1 \cdot 3$
- $(2 + 5)4$
- $[8(2) - 4^2] + 7(4)$
- $\frac{11 - 8}{1 + 7 \cdot 2}$
- $\frac{(4 \cdot 3)^2}{9 + 3}$

**Example 4** Evaluate each expression if  $a = 4$ ,  $b = 6$ , and  $c = 8$ .

- $8b - a$
- $2a + (b^2 \div 3)$
- $\frac{b(9 - c)}{a^2}$

**Example 5** 13. **BOOKS** Akira bought one new book for \$20 and three used books for \$4.95 each. Write and evaluate an expression to find how much money the books cost.

14. **MP ORGANIZE IDEAS** Koto purchased food for herself and her friends. She bought 4 cheeseburgers for \$3.99 each, 3 French fries for \$1.79 each, and 4 drinks for \$5.16. Write and evaluate an expression to find how much the food cost.

**Examples 1-3** Evaluate each expression.

15.  $7^2$

16.  $14^3$

17.  $2^6$

18.  $35 - 3 \cdot 8$

19.  $18 \div 9 + 2 \cdot 6$

20.  $10 + 8^3 \div 16$

21.  $24 \div 6 + 2^3 \cdot 4$

22.  $(11 \cdot 7) - 9 \cdot 8$

23.  $29 - 3(9 - 4)$

24.  $(12 - 6) \cdot 5^2$

25.  $3^5 - (1 + 10^2)$

26.  $108 \div [3(9 + 3^2)]$

27.  $[(6^3 - 9) \div 23]4$

28.  $\frac{8 + 3^3}{12 - 7}$

29.  $\frac{(1 + 6)^9}{5^2 - 4}$

**Example 4** Evaluate each expression if  $g = 2$ ,  $r = 3$ , and  $t = 11$ .

30.  $g + 6t$

31.  $7 - gr$

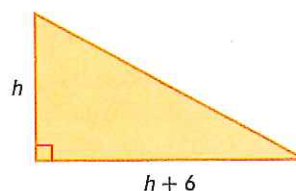
32.  $r^2 + (g^3 - 8)^5$

33.  $(2t + 3g) \div 4$

34.  $t^2 + 8rt + r^2$

35.  $3g(g + r)^2 - 1$

**Example 5** 36. **GEOMETRY** Write an algebraic expression to represent the area of the triangle. Then evaluate it to find the area when  $h = 12$  inches.



37. **TURTLES** In 1996, there were 20 Kemp's Ridley sea turtle nests off the coast of Texas. The number had increased by 190 in 2012. Write and evaluate an expression to find the number of turtle nests in 2012.

38. **MP APPLY MATH** Kamilah works at the University of Texas at Austin athletic ticket office. If  $a$  represents a Club ticket,  $b$  represents a Tier 1 ticket, and  $c$  represents a Tier 2 ticket, interpret and then evaluate the following expressions.

a.  $45b$

b.  $15a + 35c$

c.  $6a + 11b + 22c$

Texas Longhorns Football Season Ticket Prices	
Club	\$555
Tier 1	\$405
Tier 2	\$395

Source: Texas Sports

Evaluate each expression.

39.  $4^2$

40.  $12^3$

41.  $3^6$

42.  $11^5$

43.  $(3 - 4^2)^2 + 8$

44.  $23 - 2(17 + 3^3)$

45.  $3[4 - 8 + 4^2(2 + 5)]$

46.  $\frac{2 \cdot 8^2 - 2^2 \cdot 8}{2 \cdot 8}$

47.  $25 + \left[ (16 - 3 \cdot 5) + \frac{12 + 3}{5} \right]$

48.  $7^3 - \frac{2}{3}(13 \cdot 6 + 9)4$

Evaluate each expression if  $a = 8$ ,  $b = 4$ , and  $c = 16$ .

49.  $a^2bc - b^2$

50.  $\frac{c^2}{b^2} + \frac{b^2}{a^2}$

51.  $\frac{2b + 3c^2}{4a^2 - 2b}$

52.  $\frac{3ab + c^2}{a}$

53.  $\left(\frac{a}{b}\right)^2 - \frac{c}{a - b}$

54.  $\frac{2a - b^2}{ab} + \frac{c - a}{b^2}$

55. **SALES** One day, 28 small and 12 large merchant spaces were rented. Another day, 30 small and 15 large spaces were rented. Write and evaluate an expression to show the total rent collected.

**THE FLEA MARKET**

**MERCHANT SPACE RENTALS**

Small space \$15.00/day

Large space \$20.00/day

Open Daily from 9:00-6:00

56. **SHOPPING** Evelina is shopping for back-to-school clothes. She bought 3 skirts, 2 pairs of jeans, and 4 sweaters. Write and evaluate an expression to find how much she spent, not including sales tax.

Clothing	
skirt	\$35.99
jeans	\$49.99
sweater	\$32.99

57. **PYRAMIDS** The pyramid at the Louvre has a square base with a side of 35.42 meters and a height of 21.64 meters. The Great Pyramid in Egypt has a square base with a side of 230 meters and a height of 146.5 meters. The expression for the volume of a pyramid is  $\frac{1}{3}Bh$ , where  $B$  is the area of the base and  $h$  is the height.
- Draw both pyramids and label the dimensions.
  - Write a verbal expression for the difference in volume of the two pyramids.
  - Write an algebraic expression for the difference in volume of the two pyramids. Find the difference in volume.
58. **FINANCIAL LITERACY** Ginger is determining her monthly expenses. She has monthly rent  $r$ , monthly utilities  $u$ , weekly food expense  $f$ , and weekly auto expense  $a$ . Assume there are 4 weeks in a month.
- Write an algebraic expression to represent her spending in one month.
  - Suppose her monthly rent is \$550, her monthly utilities are \$115, her weekly food expenses are \$75, and her weekly auto expenses are \$125. Determine her total monthly expenses?

### H.O.T. Problems Use Higher-Order Thinking Skills

59. **ERROR ANALYSIS** Tara and Curtis are simplifying  $[4(10) - 3^2] + 6(4)$ . Is either of them correct? Explain your reasoning.

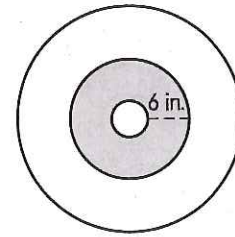
Tara	Curtis
$[4(10) - 3^2] + 6(4)$	$[4(10) - 3^2] + 6(4)$
$= [4(10) - 9] + 6(4)$	$= [4(10) - 9] + 6(4)$
$= 4(1) + 6(4)$	$= (40 - 9) + 6(4)$
$= 4 + 6(4)$	$= 31 + 6(4)$
$= 4 + 24$	$= 31 + 24$
$= 28$	$= 55$

60. **MP ANALYZE RELATIONSHIPS** Explain how to evaluate  $a[(b - c) \div d] - f$  if you were given values for  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $f$ . How would you evaluate the expression differently if the expression was  $a \cdot b - c \div d - f$ ?
61. **MP PROBLEM SOLVING** Write an expression using the whole numbers 1 to 5 using all five digits and addition and/or subtraction to create a numeric expression with a value of 3.
62. **MP ORGANIZE IDEAS** Write an expression that uses exponents, at least three different operations, and two sets of parentheses. Explain the steps you would take to evaluate the expression.
63. **WRITING IN MATH** Choose a geometric formula and explain how the order of operations applies when using the formula.
64. **WRITING IN MATH** Equivalent expressions have the same value. Are the expressions  $(30 + 17) \times 10$  and  $10 \times 30 + 10 \times 17$  equivalent? Explain why or why not.

## Example

TEKS A.5(A) MP A.1(E)

The smallest circle in the figure has a radius of 2 inches. Which equation gives the area  $A$  of the shaded part of the target? Recall that  $A = \pi r^2$ .



- A  $A = 4\pi$
- B  $A = 12\pi$
- C  $A = 60\pi$
- D  $A = 68\pi$

The diagram includes 3 circles. Subtract the area of the smallest circle from the area of the middle circle.

Area of middle circle – area of smallest circle = area of shaded region

$$\pi(6 + 2)^2$$

Write an expression for the area of the middle circle.

$$\pi(2)^2$$

Write an expression for the area of the smallest circle.

$$A = \pi(6 + 2)^2 - \pi(2)^2$$

Write an equation for the shaded area.

$$= \pi(8)^2 - \pi(2)^2$$

Evaluate inside parentheses.

$$= 64\pi - 4\pi$$

Evaluate powers.

$$= 60\pi$$

Subtract.

The correct answer is choice C.

65. Edgar buys  $a$  apples,  $b$  bananas, and  $c$  cantaloupes at the farmer's market. The prices at the market are shown in the table.

Fruit	Price Each
Apples	\$0.50
Bananas	\$0.20
Cantaloupes	\$1.50

Edgar has a coupon for a free apple. If he gets 5 apples, 4 bananas, and 2 cantaloupes, how much money does he spend? TEKS A.5(A)

MP A.1(E)

- A \$2.20
- B \$4.50
- C \$5.80
- D \$6.30

66. GRIDDABLE Evaluate  $27 \div 3 + (12 - 4)$ .

TEKS A.5(A) MP A.1(F)

67. Maya has one bran muffin, 16 ounces of orange juice, 3 ounces of sunflower seeds, 2 slices of turkey, and half of a cup of spinach.

Food	Protein (g)
bran muffin (1)	3
orange juice (8 oz)	2
sunflower seeds (1 oz)	2
turkey (1 slice)	12
spinach (1 cup)	5

Find the total number of grams of protein she consumed. TEKS A.5(A) MP A.1(A), A.1(E)

- F 24
- G 37.5
- H 39.5
- J 42



**Then**

- You used the order of operations to simplify expressions.

**Now**

- 1 Recognize the properties of equality and identity properties.
- 2 Recognize the Commutative and Associative Properties.

**Why?**

- Natalie lives 32 miles away from the mall. The distance from her house to the mall is the same as the distance from the mall to her house. This is an example of the Reflexive Property.



**Targeted TEKS**

**Preparation for A.5(A)**  
Solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.



**Mathematical Processes**

**A.1(E)** Create and use representations to organize, record, and communicate mathematical ideas.

**A.1(G)** Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.



**New Vocabulary**

- equivalent expressions
- additive identity
- multiplicative identity
- multiplicative inverse
- reciprocal

**1 Properties of Equality and Identity** The expressions  $4k + 8k$  and  $12k$  are called **equivalent expressions** because they represent the same number. The properties below allow you to write an equivalent expression for a given expression.

KeyConcept Properties of Equality			
Property	Words	Symbols	Examples
Reflexive Property	Any quantity is equal to itself.	For any number $a$ , $a = a$ .	$5 = 5$ $4 + 7 = 4 + 7$
Symmetric Property	If one quantity equals a second quantity, then the second quantity equals the first.	For any numbers $a$ and $b$ , if $a = b$ , then $b = a$ .	If $8 = 2 + 6$ , then $2 + 6 = 8$ .
Transitive Property	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.	For any numbers $a$ , $b$ , and $c$ , if $a = b$ and $b = c$ , then $a = c$ .	If $6 + 9 = 3 + 12$ and $3 + 12 = 15$ , then $6 + 9 = 15$ .
Substitution Property	A quantity may be substituted for its equal in any expression.	If $a = b$ , then $a$ may be replaced by $b$ in any expression.	If $n = 11$ , then $4n = 4 \cdot 11$

The sum of any number and 0 is equal to the number. Thus, 0 is called the **additive identity**.

KeyConcept Addition Properties			
Property	Words	Symbols	Examples
Additive Identity	For any number $a$ , the sum of $a$ and 0 is $a$ .	$a + 0 = 0 + a = a$	$2 + 0 = 2$ $0 + 2 = 2$
Additive Inverse	A number and its opposite are additive inverses of each other.	$a + (-a) = 0$	$3 + (-3) = 0$ $4 - 4 = 0$

There are also special properties associated with multiplication. Consider the following equations.

$$4 \cdot n = 4$$

The solution of the equation is 1. Since the product of any number and 1 is equal to the number, 1 is called the **multiplicative identity**.

$$6 \cdot m = 0$$

The solution of the equation is 0. The product of any number and 0 is equal to 0. This is called the **Multiplicative Property of Zero**.

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. Zero has no reciprocal because any number times 0 is 0.

### StudyTip

**Using Your Text** Look for Key Concepts to learn important properties, definitions, and concepts.

Key Concept Multiplication Properties			
Property	Words	Symbols	Examples
Multiplicative Identity	For any number $a$ , the product of $a$ and 1 is $a$ .	$a \cdot 1 = a$ $1 \cdot a = a$	$14 \cdot 1 = 14$ $1 \cdot 14 = 14$
Multiplicative Property of Zero	For any number $a$ , the product of $a$ and 0 is 0.	$a \cdot 0 = 0$ $0 \cdot a = 0$	$9 \cdot 0 = 0$ $0 \cdot 9 = 0$
Multiplicative Inverse	For every number $\frac{a}{b}$ , where $a, b \neq 0$ , there is exactly one number $\frac{b}{a}$ such that the product of $\frac{a}{b}$ and $\frac{b}{a}$ is 1.	$\frac{a}{b} \cdot \frac{b}{a} = 1$ $\frac{b}{a} \cdot \frac{a}{b} = 1$	$\frac{4}{5} \cdot \frac{5}{4} = \frac{20}{20}$ or 1 $\frac{5}{4} \cdot \frac{4}{5} = \frac{20}{20}$ or 1

### StudyTip

**MP Justify Arguments** As you evaluate an expression, you are constructing an argument using stated assumptions, definitions, and previously established results. The properties of numbers are valid reasons for steps in the argument.

### Example 1 Evaluate Using Properties

Evaluate  $7(4 - 3) - 1 + 5 \cdot \frac{1}{5}$ . Name the property used in each step.

$$\begin{aligned}
 7(4 - 3) - 1 + 5 \cdot \frac{1}{5} &= 7(1) - 1 + 5 \cdot \frac{1}{5} && \text{Substitution: } 4 - 3 = 1 \\
 &= 7 - 1 + 5 \cdot \frac{1}{5} && \text{Multiplicative Identity: } 7 \cdot 1 = 7 \\
 &= 7 - 1 + 1 && \text{Multiplicative Inverse: } 5 \cdot \frac{1}{5} = 1 \\
 &= 6 + 1 && \text{Substitution: } 7 - 1 = 6 \\
 &= 7 && \text{Substitution: } 6 + 1 = 7
 \end{aligned}$$

### Guided Practice

Name the property used in each step.

**1A.**  $2 \cdot 3 + (4 \cdot 2 - 8)$

$$\begin{aligned}
 &= 2 \cdot 3 + (8 - 8) \quad ? \\
 &= 2 \cdot 3 + (0) \quad ? \\
 &= 6 + 0 \quad ? \\
 &= 6 \quad ?
 \end{aligned}$$

**1B.**  $7 \cdot \frac{1}{7} + 6(15 \div 3 - 5)$

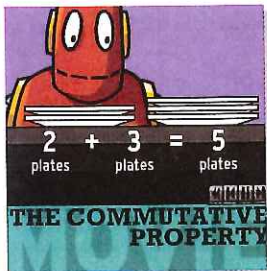
$$\begin{aligned}
 &= 7 \cdot \frac{1}{7} + 6(5 - 5) \quad ? \\
 &= 7 \cdot \frac{1}{7} + 6(0) \quad ? \\
 &= 1 + 6(0) \quad ? \\
 &= 1 + 0 \quad ? \\
 &= 1 \quad ?
 \end{aligned}$$



## Go Online!



Look for the blue **Watch** icons for concepts that have video animations. Log into ConnectED to see them.



**2 Use Commutative and Associative Properties** Nikki walks 2 blocks to her friend Sierra's house. They walk another 4 blocks to school. At the end of the day, Nikki and Sierra walk back to Sierra's house, and then Nikki walks home.

$$\begin{array}{ccc} \text{The distance from} & & \text{the distance from the} \\ \text{Nikki's house to school} & \text{equals} & \text{school to Nikki's house.} \\ 2 + 4 & = & 4 + 2 \end{array}$$

This is an example of the **Commutative Property** for addition.

### KeyConcept Commutative Property

<b>Words</b>	The order in which you add or multiply numbers does not change their sum or product.
<b>Symbols</b>	For any numbers $a$ and $b$ , $a + b = b + a$ and $a \cdot b = b \cdot a$ .
<b>Examples</b>	$4 + 8 = 8 + 4$ $7 \cdot 11 = 11 \cdot 7$

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the **Associative Property**.

### KeyConcept Associative Property

<b>Words</b>	The way you group three or more numbers when adding or multiplying does not change their sum or product.
<b>Symbols</b>	For any numbers $a$ , $b$ , and $c$ , $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$ .
<b>Examples</b>	$(3 + 5) + 7 = 3 + (5 + 7)$ $(2 \cdot 6) \cdot 9 = 2 \cdot (6 \cdot 9)$



### Real-WorldLink

A birthday party may cost about \$200 depending on the number of people invited.

Source: Family Corner

### Real-World Example 2 Apply Properties of Numbers

**PARTY PLANNING** Eric makes a list of items that he needs to buy for a party and their costs. Find the total cost of these items.

$$\begin{array}{ccccccc} \text{Balloons} & & \text{Decorations} & & \text{Food} & & \text{Beverages} \\ 9.75 & + & 18.50 & + & 53.25 & + & 22.50 \end{array}$$

$$\begin{aligned} &= 9.75 + 53.25 + 18.50 + 22.50 && \text{Commutative (+)} \\ &= (9.75 + 53.25) + (18.50 + 22.50) && \text{Associative (+)} \\ &= 63.00 + 41.00 && \text{Substitution} \\ &= 104.00 && \text{Substitution} \end{aligned}$$

The total cost is \$104.00

Party Supplies	
Item	Cost (\$)
balloons	9.75
decorations	18.50
food	53.25
beverages	22.50

### GuidedPractice

**2. FURNITURE** Rafael is buying furnishings for his first apartment. He buys a couch for \$450, lamps for \$55.50, a rug for \$43.50, and a table for \$75. Find the total cost of these items.



### Example 3 Use Multiplication Properties

Evaluate  $5 \cdot 7 \cdot 4 \cdot 2$  using the properties of numbers. Name the property used in each step.

$$\begin{aligned}
 5 \cdot 7 \cdot 4 \cdot 2 &= 5 \cdot 2 \cdot 7 \cdot 4 && \text{Commutative } (\times) \\
 &= (5 \cdot 2) \cdot (7 \cdot 4) && \text{Associative } (\times) \\
 &= 10 \cdot 28 && \text{Substitution} \\
 &= 280 && \text{Substitution}
 \end{aligned}$$

#### Guided Practice

Evaluate each expression using the properties of numbers. Name the property used in each step.

3A.  $2.9 \cdot 4 \cdot 10$

3B.  $\frac{5}{3} \cdot 25 \cdot 3 \cdot 2$

### Check Your Understanding



= Step-by-Step Solutions begin on page R13.



Go Online! for a Self-Check Quiz

**Example 1** Evaluate each expression. Name the property used in each step.

1.  $(1 \div 5)5 \cdot 14$

2.  $6 + 4(19 - 15)$

3.  $5(14 - 5) + 6(3 + 7)$

4. **FINANCIAL LITERACY** Carolyn has 9 quarters, 4 dimes, 7 nickels, and 2 pennies, which can be represented as  $9(25) + 4(10) + 7(5) + 2$ . Evaluate the expression to find how much money she has. Name the property used in each step.

**Examples 2-3** Evaluate each expression using the properties of numbers. Name the property used in each step.

5.  $23 + 42 + 37$

6.  $2.75 + 3.5 + 4.25 + 1.5$

7.  $3 \cdot 7 \cdot 10 \cdot 2$

8.  $\frac{1}{4} \cdot 24 \cdot \frac{2}{3}$

### Practice and Problem Solving

Extra Practice is on page R1.

**Example 1** Evaluate each expression. Name the property used in each step.

9.  $3(22 - 3 \cdot 7)$

10.  $7 + (9 - 3^2)$

11.  $\frac{3}{4} [4 \div (7 - 4)]$

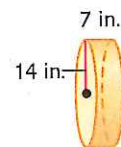
12.  $[3 \div (2 \cdot 1)] \frac{2}{3}$

13.  $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$

14.  $6 \cdot \frac{1}{6} + 5(12 \div 4 - 3)$

**Example 2**

15. **GEOMETRY** The expression  $2 \cdot \frac{22}{7} \cdot 14^2 + 2 \cdot \frac{22}{7} \cdot 14 \cdot 7$  represents the approximate surface area of the cylinder at the right. Evaluate this expression to find the approximate surface area. Name the property used in each step.



16. **MP ORGANIZE IDEAS** A traveler checks into a hotel on Friday and checks out the following Tuesday morning. Use the table to find the total cost of the room including tax.

Hotel Rates Per Day		
Day	Room Charge	Sales Tax
Monday-Friday	\$99	\$12.87
Saturday-Sunday	\$87	\$11.31

**Examples 2-3** Evaluate each expression using properties of numbers. Name the property used in each step.

17.  $25 + 14 + 15 + 36$

18.  $11 + 7 + 5 + 13$

19.  $3\frac{2}{3} + 4 + 5\frac{1}{3}$

20.  $4\frac{4}{9} + 7\frac{2}{9}$

21.  $4.3 + 2.4 + 3.6 + 9.7$

22.  $3.25 + 2.2 + 5.4 + 10.75$

23.  $12 \cdot 2 \cdot 6 \cdot 5$

24.  $2 \cdot 8 \cdot 10 \cdot 2$

25.  $0.2 \cdot 4.6 \cdot 5$

26.  $3.5 \cdot 3 \cdot 6$

27.  $1\frac{5}{6} \cdot 24 \cdot 3\frac{1}{11}$

28.  $2\frac{3}{4} \cdot 1\frac{1}{8} \cdot 32$

29. **SCUBA DIVING** The sign shows the equipment rented or sold by a scuba diving store.

- Write two expressions to represent the total sales to rent 2 wet suits, 3 air tanks, 2 dive flags, and selling 5 underwater cameras.
- What are the total sales?

30. **COOKIES** Bobby baked 2 dozen chocolate chip cookies, 3 dozen sugar cookies, and a dozen oatmeal raisin cookies. How many total cookies did he bake?

THE DEEP SCUBA SUPPLIES	
SPECIALS	
Underwater Camera	\$18.99
RENTALS	
Air Tanks	\$15.50
Wet Suit	\$17.25
Dive Flag	\$ 5.00



Evaluate each expression if  $a = -1$ ,  $b = 4$ , and  $c = 6$ .

31.  $4a + 9b - 2c$

32.  $-10c + 3a + a$

33.  $a - b + 5a - 2b$

34.  $8a + 5b - 11a - 7b$

35.  $3c^2 + 2c + 2c^2$

36.  $3a - 4a^2 + 2a$

37. **FOOTBALL** A football team is on the 35-yard line. The quarterback is sacked at the line of scrimmage. The team gains 0 yards, so they are still at the 35-yard line. Which identity or property does this represent? Explain.

Find the value of  $x$ . Then name the property used.

38.  $8 = 8 + x$

39.  $3.2 + x = 3.2$

40.  $10x = 10$

41.  $\frac{1}{2} \cdot x = \frac{1}{2} \cdot 7$

42.  $x + 0 = 5$

43.  $1 \cdot x = 3$

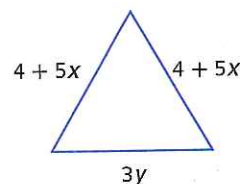
44.  $5 \cdot \frac{1}{5} = x$

45.  $2 + 8 = 8 + x$

46.  $x + \frac{3}{4} = 3 + \frac{3}{4}$

47.  $\frac{1}{3} \cdot x = 1$

48. **GEOMETRY** Write an expression to represent the perimeter of the triangle. Then find the perimeter if  $x = 2$  and  $y = 7$ .



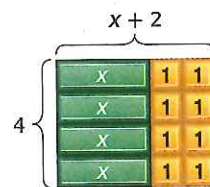
49. **SPORTS** Tickets to a baseball game cost \$35 each plus a \$5.50 handling charge per ticket. If Sharon has a coupon for \$10 off and orders 4 tickets, how much will she be charged?

50. **MP APPLY MATH** The table shows prices on children's clothing.

- Interpret the expression  $5(17.99) + 2(11.99) + 7(14.99)$ .
- Write and evaluate three different expressions that represent 8 pairs of shorts and 8 tops.
- If you buy 8 shorts and 8 tops, you receive a discount of 15%. Find the greatest and least amount of money you can spend on the 16 items at the sale.

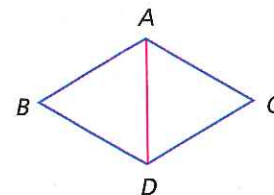
Shorts	Polos	T-shirts
\$16.99	\$17.99	\$15.99
\$14.99	\$13.99	\$11.99

51. **MULTI-STEP** George is designing ledges for the octagonal (8-sided) gazebo that his brother is building. All of the sides are equal in length, and each ledge needs to be 18 inches shorter than the sides
- What is the minimum length of wood George should purchase if his brother decides that the perimeter of the gazebo will be 64 feet?
  - What was your solution process?
  - What assumptions did you make?



52. **MP MULTIPLE REPRESENTATIONS** You can use algebra tiles to model and explore algebraic expressions. The rectangular tile has an area of  $x$ , with dimensions 1 by  $x$ . The small square tile has an area of 1, with dimensions 1 by 1.
- Concrete** Make a rectangle with algebra tiles to model the expression  $4(x + 2)$  as shown above. What are the dimensions of this rectangle? What is its area?
  - Analytical** What are the areas of the green region and of the yellow region?
  - Verbal** Complete this statement:  $4(x + 2) = ?$ . Write a convincing argument to justify your statement.

53. **GEOMETRY** A **proof** is an argument in which each statement you make is supported by a true statement. It is given that  $\overline{AB} \cong \overline{CD}$ ,  $\overline{AB} \cong \overline{BD}$ , and  $\overline{AB} \cong \overline{AC}$ . Pedro wants to prove  $\triangle ADB \cong \triangle ADC$ . To do this, he must show that  $\overline{AD} \cong \overline{AD}$ ,  $\overline{AB} \cong \overline{DC}$  and  $\overline{BD} \cong \overline{AC}$ .



- Copy the figure and label  $\overline{AB} \cong \overline{CD}$ ,  $\overline{AB} \cong \overline{BD}$ , and  $\overline{AB} \cong \overline{AC}$ .
- Use the Reflexive and Transitive Properties to prove  $\triangle ADB \cong \triangle ADC$ .
- If  $AC$  is  $x$  centimeters, write an equation for the perimeter of  $ACDB$ .

### H.O.T. Problems

### Use Higher-Order Thinking Skills

54. **MP ORGANIZE IDEAS** Write two equations showing the Transitive Property of Equality. Justify your reasoning.
55. **MP JUSTIFY ARGUMENTS** Explain why 0 has no multiplicative inverse.
56. **MP JUSTIFY ARGUMENTS** The sum of any two whole numbers is always a whole number. So, the set of whole numbers  $\{0, 1, 2, 3, 4, \dots\}$  is said to be closed under addition. This is an example of the **Closure Property**. State whether each statement is *true* or *false*. If false, justify your reasoning.
- The set of whole numbers is closed under subtraction.
  - The set of whole numbers is closed under multiplication.
  - The set of whole numbers is closed under division.
57. **MP JUSTIFY ARGUMENTS** Does the Commutative Property *sometimes*, *always* or *never* hold for subtraction? Explain your reasoning.
58. **MP ANALYZE RELATIONSHIPS** Explain whether 1 can be an additive identity. Give an example to justify your answer.
59. **WHICH ONE DOESN'T BELONG?** Identify the equation that does not belong with the other three. Explain your reasoning.

$$x + 12 = 12 + x$$

$$7h = h \cdot 7$$

$$1 + a = a + 1$$

$$(2j)k = 2(jk)$$

60. **WRITING IN MATH** Determine whether the Commutative Property applies to division. Justify your answer.



## Example

TEKS A.5(A) MP A.1(D), A.1(G)

Renee wants to solve the equation  $\frac{3}{4}b = 6$ . To do this, she will use the Multiplicative Inverse Property.

Which of the following equations best illustrates this property?

- A  $8 - 7 = 1$
- B  $a = a \cdot 1$
- C  $b \div b = 1$  ( $b \neq 0$ )
- D  $\frac{c}{d} \cdot \frac{d}{c} = 1$  ( $c \neq 0, d \neq 0$ )

Look at the pattern shown in each answer choice:

The equation  $8 - 7 = 1$  represents subtraction.

The equation  $a = a \cdot 1$  represents the property that states any number can be expressed as the product of itself and 1. That is the Multiplicative Identity Property.

The equation  $b \div b = 1$  represents the property that states when any number (except zero) is divided by itself, the quotient is 1.

The equation  $\frac{c}{d} \cdot \frac{d}{c} = 1$  represents the property that states multiplying a fraction by its reciprocal is 1. The reciprocal is found by switching the numerator and the denominator. That is the Multiplicative Inverse Property.

The correct answer is choice D.

61. Abassi will use the Additive Identity Property to solve an equation. Which of the following best illustrates the Additive Identity Property?

TEKS A.5(A) MP A.1(D), A.1(G)

- A  $a \cdot 1 = a$
- B  $b + 0 = b$
- C  $c + (-c) = 0$
- D  $d + 1 = d + 1$

62. When a number is tripled, its value increases by 10. What is the original number? TEKS A.2(C),

A.5(A) MP A.1(B)

- F 5
- G 10
- H 15
- J 30

63. Which property justifies rewriting the equation

$$\frac{1}{6} \cdot 6 + z = 8 \text{ as } 1 + z = 8? \text{ TEKS A.5(A) MP A.1(G)}$$

- A Additive Identity Property
- B Multiplicative Identity Property
- C Multiplicative Inverse Property
- D Substitution

64. A company creates mobile apps for a smartphone. When the app was free, they had 880 downloads. After the price was set to \$0.99, they had  $d$  downloads. The company receives \$0.70 in revenue for each app that is sold for \$0.99. Which equation gives the average revenue  $R$  for all downloads of this app? TEKS A.2(C), A.5(A) MP A.1(A)

- F  $R = \frac{0.7d}{880 + d}$
- G  $R = 0.7(880 - d)$
- H  $R = 0.7d$
- J  $R = \frac{0.7}{880 + d}$



All measurements taken in the real world are approximations. The greater the care with which a measurement is taken, the more accurate it will be. **Accuracy** refers to how close a measured value comes to the actual or desired value. For example, a fraction is more accurate than a rounded decimal.



### Activity 1 When Is Close Good Enough?

**Work cooperatively. Measure your desktop. Record your results in centimeters, in meters, and in millimeters.**

#### Analyze the Results

1. Did you round to the nearest whole measure? If so, when?
2. Did you round to the nearest half, tenth, or smaller? If so, when?
3. Which unit of measure was the most appropriate for this task?
4. Which unit of measure was the most accurate?

Deciding where to round a measurement depends on how the measurement will be used. But calculations should not be carried out to greater accuracy than that of the original data.

### Activity 2 Decide Where to Round

- a. **Elan has \$13 that he wants to divide among his 6 nephews. When he types  $13 \div 6$  into his calculator, the number that appears is 2.16666667. Where should Elan round?**

Since Elan is rounding money, the smallest increment is a penny, so round to the hundredths place. This will give him 2.17, and  $\$2.17 \times 6 = \$13.02$ . Elan will be two pennies short, so round to \$2.16. Since  $\$2.16 \times 6 = \$12.96$ , Elan can give each of his nephews \$2.16.

- b. **Dante's mother brings him a dozen cookies, but before she leaves she eats one and tells Dante he has to share with his two sisters. Dante types  $11 \div 3$  into his calculator and gets 3.66666667. Where should Dante round?**

After each sibling receives 3 cookies, there are two cookies left. In this case, it is more accurate to convert the decimal portion to a fraction and give each sibling  $\frac{2}{3}$  of a cookie.

- c. **Eva measures the dimensions of a box as 8.7, 9.52, and 3.16 inches. She multiplies these three numbers to find the measure of the volume. The result shown on her calculator is 261.72384. Where should Eva round?**

Eva should round to the tenths place, 261.7, because she was only accurate to the tenths place with one of her measures.

### Exercises

5. Jessica wants to divide \$23 six ways. Her calculator shows 3.83333333. Where should she round?
6. Ms. Harris wants to share 2 pizzas among 6 people. Her calculator shows 0.33333333. Where should she round?
7. The measurements of an aquarium are 12.9, 7.67, and 4.11 inches. The measure of the volume is given by the product 406.65573. Where should the number be rounded?

For most real-world measurements, a decision must be made on the level of accuracy needed or desired.

### Activity 3 Find an Appropriate Level of Accuracy

- a. Jon needs to buy a shade for the window opening shown, but the shades are only available in whole inch increments. What size shade should he buy?

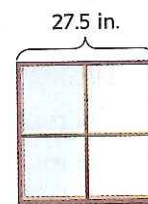
He should buy the 27-inch shade because it will be enough to cover the glass.

- b. Tom is buying flea medicine for his dog. The amount of medicine depends on the dog's weight. The medicine is available in packages that vary by 10 dog pounds. How accurate does Tom need to be to buy the correct medicine?

He needs to be accurate to within 10 pounds.

- c. Tyrone is building a jet engine. How accurate do you think he needs to be with his measurements?

He needs to be very accurate, perhaps to the thousandth of an inch.



### Exercises

8. Matt's table is missing a leg. He wants to cut a piece of wood to replace the leg. How accurate do you think he needs to be with his measurements?

For each situation, determine where the rounding should occur and give the rounded answer.

9. Sam wants to divide \$111 seven ways. His calculator shows 15.85714286.
10. Kiri wants to share 3 pies among 11 people. Her calculator shows 0.2727272727.
11. Evan's calculator gives him the volume of his soccer ball as 137.2582774. Evan measured the radius of the ball to be 3.2 inches.

For each situation, determine the level of accuracy needed. Explain.

12. You are estimating the length of your school's basketball court. Which unit of measure should you use: 1 foot, 1 inch, or  $\frac{1}{16}$  inch?
13. You are estimating the height of a small child. Which unit of measure should you use: 1 foot, 1 inch, or  $\frac{1}{16}$  inch?
14. **TRAVEL** Curt is measuring the driving distance from one city to another. How accurate do you think he needs to be with his measurement?
15. **MEDICINE** A nurse is administering medicine to a patient based on his weight. How accurate do you think she needs to be with her measurements?



## Then

- You explored Associative and Commutative Properties.

## Now

- Use the Distributive Property to evaluate expressions.
- Use the Distributive Property to simplify expressions.

## Why?

- John burns approximately 420 Calories per hour by surfing. The chart below shows the time he spent surfing in one week.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Time (h)	1	$\frac{1}{2}$	0	1	0	2	$2\frac{1}{2}$

To determine the total number of Calories that he burned surfing that week, you can use the Distributive Property.



## Targeted TEKS

**A.10(D)** Rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.



## Mathematical Processes

**A.1(A)** Apply mathematics to problems arising in everyday life, society, and the workplace.

**A.1(E)** Create and use representations to organize, record, and communicate mathematical ideas.



## New Vocabulary

like terms  
simplest form  
coefficient

**1 Evaluate Expressions** There are two methods you could use to calculate the number of Calories John burned surfing. You could find the total time spent surfing and then multiply by the Calories burned per hour. Or you could find the number of Calories burned each day and then add to find the total.

**Method 1** Rate Times Total Time

$$\begin{aligned} 420\left(1 + \frac{1}{2} + 1 + 2 + 2\frac{1}{2}\right) \\ = 420(7) \\ = 2940 \end{aligned}$$

**Method 2** Sum of Daily Calories Burned

$$\begin{aligned} 420(1) + 420\left(\frac{1}{2}\right) + 420(1) + 420(2) + 420\left(2\frac{1}{2}\right) \\ = 420 + 210 + 420 + 840 + 1050 \\ = 2940 \end{aligned}$$

Either method gives the same total of 2940 Calories burned. This is an example of the **Distributive Property**.

**Key Concept** Distributive Property

Symbol

For any numbers  $a$ ,  $b$ , and  $c$ ,  
 $a(b + c) = ab + ac$  and  $(b + c)a = ba + ca$  and  
 $a(b - c) = ab - ac$  and  $(b - c)a = ba - ca$ .

Examples

$$\begin{aligned} 3(2 + 5) &= 3 \cdot 2 + 3 \cdot 5 \\ 3(7) &= 6 + 15 \\ 21 &= 21 \end{aligned}$$

$$\begin{aligned} 4(9 - 7) &= 4 \cdot 9 - 4 \cdot 7 \\ 4(2) &= 36 - 28 \\ 8 &= 8 \end{aligned}$$

The Symmetric Property of Equality allows the Distributive Property to be written as follows.

$$\text{If } a(b + c) = ab + ac, \text{ then } ab + ac = a(b + c).$$





### Real-World Example 1 Distribute Over Addition

**SPORTS** A group of 7 adults and 6 children are going to a baseball game at Rice's Reckling Park. Use the Distributive Property to write and evaluate an expression for the total ticket cost.

**Analyze** You need to find the cost of each ticket and then find the total cost.

**Formulate**  $7 + 6$  or 13 people are going to the game, so the tickets are \$8 each.

**Determine** Write an expression that shows the product of the cost of each ticket and the sum of adult tickets and children's tickets.

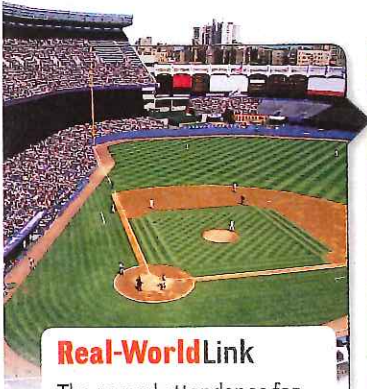
$$\begin{aligned} 8(7 + 6) &= 8(7) + 8(6) && \text{Distributive Property} \\ &= 56 + 48 && \text{Multiply.} \\ &= 104 && \text{Add.} \end{aligned}$$

The total cost is \$104.

**Justify** The total number of tickets needed is 13 and they cost \$8 each. Multiply 13 by 8 to get 104. Therefore, the total cost of tickets is \$104.

**Evaluate** The solution method used to justify our answer is more efficient than distributing over addition. Choosing the best solution method is an important problem solving skill as it reduces time and errors.

Rice Owls Baseball Tickets	
Ticket	Cost (\$)
Adult Single Game	16
Children Single Game (12 and under)	14
Groups of 10 or more Single Game	8
Senior Single Game (65 and over)	10



#### Real-World Link

The record attendance for a single baseball game was set in 1959. There were 92,706 spectators at a game between the Los Angeles Dodgers and the Chicago White Sox.

Source: Baseball Almanac

#### StudyTip

##### MP Organize Ideas and

**Apply Math** The five-step problem solving plan is a tool for making sense of any problem. When making and executing your plan, continually ask yourself, "Does this make sense?" Monitor and evaluate your progress and change course if necessary.

#### Guided Practice

- SPORTS** A group of 3 adults, an 11-year old, and 2 children under 10 years old are going to a baseball game. Write and evaluate an expression to determine the cost of tickets for the group.

You can use the Distributive Property to make mental math easier.



#### Example 2 Mental Math

Use the Distributive Property to rewrite  $7 \cdot 49$ . Then evaluate.

$$\begin{aligned} 7 \cdot 49 &= 7(50 - 1) && \text{Think: } 49 = 50 - 1 \\ &= 7(50) - 7(1) && \text{Distributive Property} \\ &= 350 - 7 && \text{Multiply.} \\ &= 343 && \text{Subtract.} \end{aligned}$$

#### Guided Practice

Use the Distributive Property to rewrite each expression. Then evaluate.

2A.  $304(15)$

2B.  $44 \cdot 2\frac{1}{2}$

2C.  $210(5)$

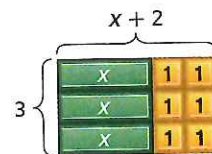
2D.  $52(17)$

**2 Simplify Expressions** You can use algebra tiles to investigate how the Distributive Property relates to algebraic expressions.

**Problem-Solving Tip**

**Make a Model** It can be helpful to visualize a problem using algebra tiles or folded paper.

The rectangle at the right has 3  $x$ -tiles and 6 1-tiles. The area of the rectangle is  $x + 1 + 1 + x + 1 + 1 + x + 1 + 1$  or  $3x + 6$ . Therefore,  $3(x + 2) = 3x + 6$ .



TEKS A.10(D)

**Example 3 Algebraic Expressions**

Rewrite each expression using the Distributive Property. Then simplify.

a.  $7(3w - 5)$

$$\begin{aligned} 7(3w - 5) &= 7 \cdot 3w - 7 \cdot 5 \\ &= 21w - 35 \end{aligned}$$

Distributive Property

Multiply.

b.  $(6v^2 + v - 3)4$

$$\begin{aligned} (6v^2 + v - 3)4 &= 6v^2(4) + v(4) - 3(4) \\ &= 24v^2 + 4v - 12 \end{aligned}$$

Distributive Property

Multiply.

**Guided Practice**

3A.  $(8 + 4n)2$

3B.  $-6(r + 3g - t)$

3C.  $(3x^2 - 5x + 2)(-5)$

3D.  $2(7 - 4m^2)$

**Review Vocabulary**

**term** a number, a variable, or a product or quotient of numbers and variables

**Like terms** are terms that contain the same variables, with corresponding variables having the same power.

$$5x^2 + 2x - 4$$

three terms

$$6a^2 + a^2 + 2a$$

like terms

unlike terms

The Distributive Property and the properties of equality can be used to show that  $4k + 8k = 12k$ . In this expression,  $4k$  and  $8k$  are like terms.

$$\begin{aligned} 4k + 8k &= (4 + 8)k && \text{Distributive Property} \\ &= 12k && \text{Substitution} \end{aligned}$$

An expression is in **simplest form** when it contains no like terms or parentheses.

TEKS A.10(D)

**Example 4 Combine Like Terms**

a. Simplify  $17u + 25u$ .

$$\begin{aligned} 17u + 25u &= (17 + 25)u \\ &= 42u \end{aligned}$$

Distributive Property

Substitution

b. Simplify  $6t^2 + 3t - t$ .

$$\begin{aligned} 6t^2 + 3t - t &= 6t^2 + (3 - 1)t \\ &= 6t^2 + 2t \end{aligned}$$

Distributive Property

Substitution

**Guided Practice**

Simplify each expression. If not possible, write *simplified*.

4A.  $6n - 4n$

4B.  $b^2 + 13b + 13$

4C.  $4y^3 + 2y - 8y + 5$

4D.  $7a + 4 - 6a^2 - 2a$



## Go Online!



Look for the **Tools** icons for places where the tools in the eToolkit may be useful. Log into ConnectED to use the tools.



## Example 5 Write and Simplify Expressions

Use the expression *twice the difference of  $3x$  and  $y$  increased by five times the sum of  $x$  and  $2y$ .*

a. Write an algebraic expression for the verbal expression.

Words	twice the difference of $3x$ and $y$	increased by	five times the sum of $x$ and $2y$
Variables	Let $x$ and $y$ represent the numbers.		
Expression	$2(3x - y)$	+	$5(x + 2y)$

b. Simplify the expression, and indicate the properties used.

$$\begin{aligned}
 2(3x - y) + 5(x + 2y) &= 2(3x) - 2(y) + 5(x) + 5(2y) && \text{Distributive Property} \\
 &= 6x - 2y + 5x + 10y && \text{Multiply.} \\
 &= 6x + 5x - 2y + 10y && \text{Commutative (+)} \\
 &= (6 + 5)x + (-2 + 10)y && \text{Distributive Property} \\
 &= 11x + 8y && \text{Substitution}
 \end{aligned}$$

## Guided Practice

5. Use the expression *5 times the difference of  $q$  squared and  $r$  plus 8 times the sum of  $3q$  and  $2r$ .*

- Write an algebraic expression for the verbal expression.
- Simplify the expression, and indicate the properties used.

The **coefficient** of a term is the numerical factor. For example, in  $6ab$ , the coefficient is 6, and in  $\frac{x^2}{3}$ , the coefficient is  $\frac{1}{3}$ . In the term  $y$ , the coefficient is 1 since  $1 \cdot y = y$  by the Multiplicative Identity Property.

## Concept Summary Properties of Numbers

The following properties are true for any numbers  $a$ ,  $b$ , and  $c$ .

Properties	Addition	Multiplication
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	0 is the identity. $a + 0 = 0 + a = a$	1 is the identity. $a \cdot 1 = 1 \cdot a = a$
Zero	—	$a \cdot 0 = 0 \cdot a = 0$
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	
Substitution	If $a = b$ , then $a$ may be substituted for $b$ .	

## StudyTip

## Compare and Contrast

Noticing the similarities and differences in the addition and multiplication properties can help you learn these terms.

## Check Your Understanding

 = Step-by-Step Solutions begin on page R13.



Go Online! for a Self-Check Quiz

**Example 1**  
**TEKS** A.10(D)

1. **PILOT** A pilot at an air show charges \$55 per passenger for rides. If 12 adults and 15 children ride in one day, write and evaluate an expression to describe the situation.

**Example 2**  
**TEKS** A.10(D)

Use the Distributive Property to rewrite each expression. Then evaluate.

2.  $14(51)$

3.  $6\frac{1}{9}(9)$

**Example 3**  
**TEKS** A.10(D)

Use the Distributive Property to rewrite each expression. Then simplify.

4.  $2(4 + t)$

5.  $(2g^2 + 9g - 3)6$

**Example 4**  
**TEKS** A.10(D)

Simplify each expression. If not possible, write *simplified*.

6.  $15m + m$

7.  $3x^3 + 5y^3 + 14$

8.  $(5m + 2m)10$

**Example 5**  
**TEKS** A.10(D)

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

9. 4 times the sum of 2 times  $x$  and six

10. one half of 4 times  $y$  plus the quantity of  $y$  and 3

## Practice and Problem Solving

Extra Practice is on page R1.

**Example 1**  
**TEKS** A.10(D)

11. **TIME MANAGEMENT** Margo uses colors to track her activities on a calendar. Red represents homework, yellow represents work, and green represents track practice. In a typical week, she has 5 red items, 3 yellow items, and 4 green items. How many activities does Margo do in 4 weeks?

12. **MP ORGANIZE IDEAS** The Red Cross is holding blood drives in two locations. In one day, Center 1 collected 715 pints and Center 2 collected 1035 pints. Write and evaluate an expression to estimate the total number of pints of blood donated over a 3-day period.

**Example 2**  
**TEKS** A.10(D)

Use the Distributive Property to rewrite each expression. Then evaluate.

13.  $(4 + 5)6$

14.  $7(13 + 12)$

15.  $6(6 - 1)$

16.  $(3 + 8)15$

17.  $14(8 - 5)$

18.  $(9 - 4)19$

19.  $4(7 - 2)$

20.  $7(2 + 1)$

21.  $7 \cdot 497$

22.  $6(525)$

23.  $36 \cdot 3\frac{1}{4}$

24.  $(4\frac{2}{7})21$

**Example 3**  
**TEKS** A.10(D)

Use the Distributive Property to rewrite each expression. Then simplify.

25.  $2(x + 4)$

26.  $(5 + n)3$

27.  $(2 - 3m^2)(-5)$

28.  $8(x^2 - 9x + 5)$

**Example 4**  
**TEKS** A.10(D)

Simplify each expression. If not possible, write *simplified*.

29.  $13r + 5r$

30.  $3x^3 - 2x^2$

31.  $7m + 7 - 5m$

32.  $5z^2 + 3z + 8z^2$

33.  $(2 - 4n)17$

34.  $11(4d + 6)$

35.  $7m + 2m + 5p + 4m$

36.  $3x + 7(3x + 4)$

37.  $4(fg + 3g) + 5g$

**Example 5**  
**TEKS** A.10(D)

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

38. the product of 5 and  $m$  squared, increased by the sum of the square of  $m$  and 5

39. 7 times the sum of  $a$  squared and  $b$  minus 4 times the sum of  $a$  squared and  $b$

40. **GEOMETRY** Find the perimeter of an isosceles triangle with side lengths of  $5 + x$ ,  $5 + x$ , and  $xy$ . Write in simplest form.
41. **GEOMETRY** A regular hexagon measures  $3x + 5$  units on each side. What is the perimeter in simplest form?

Simplify each expression.

42.  $6x + 4y + 5x$       43.  $3m + 5g + 6g + 11m$       44.  $4a + 5a^2 + 2a^2 + a^2$
45.  $5k + 3k^3 + 7k + 9k^3$       46.  $6d + 4(3d + 5)$       47.  $2(6x + 4) + 7x$

48. **FOOD** Kenji is picking up take-out food for his study group.

- a. Interpret the expression  $4(4.49) + 3(2.29) + 3(1.99) + 5(1.49)$ .
- b. How much would it cost if Kenji bought four of each item on the menu?

Menu	
Item	Cost (\$)
sandwich	4.49
cup of soup	2.29
side salad	1.99
drink	1.49

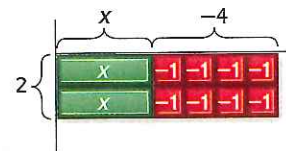
Use the Distributive Property to rewrite each expression. Then simplify.

49.  $(\frac{1}{3} - 2b)27$       50.  $4(8p + 4q - 7r)$       51.  $6(2c - cd^2 + d)$

Simplify each expression. If not possible, write *simplified*.

52.  $6x^2 + 14x - 9x$       53.  $4y^3 + 3y^3 + y^4$       54.  $a + \frac{a}{5} + \frac{2}{5}a$

55. **MP MULTIPLE REPRESENTATIONS** The area of the model is  $2(x - 4)$  or  $2x - 8$ . The expression  $2(x - 4)$  is in factored form.



- a. **Geometric** Use algebra tiles to form a rectangle with area  $2x + 6$ . Use the result to write  $2x + 6$  in factored form.
- b. **Tabular** Use algebra tiles to form rectangles to represent each area in the table. Record the factored form of each expression.
- c. **Verbal** Explain how you could find the factored form of an expression.

Area	Factored Form
$2x + 6$	
$3x + 3$	
$3x - 12$	
$5x + 10$	

TEKS A.10(D)

### H.O.T. Problems

Use Higher-Order Thinking Skills

56. **MP APPLY MATH** Use the Distributive Property to simplify  $6x^2[(3x - 4) + (4x + 2)]$ .
57. **MP ANALYZE RELATIONSHIPS** Should the Distributive Property be a property of multiplication, addition, or both? Explain your answer.
58. **WRITING IN MATH** Why is it helpful to represent verbal expressions algebraically?
59. **WRITING IN MATH** Use the data about surfing on page 25 to explain how the Distributive Property can be used to calculate quickly. Also, compare the two methods of finding the total Calories burned.



## Example

TEKS A.10(D) MP A.1(D), A.1(F)

An expression is shown below.

$$6ab^2 + 9a^2b$$

Which of the following shows an equivalent expression?

- A  $9a^2b^2$
- B  $3ab(3b + 2a)$
- C  $3ab(2b + 3a)$
- D  $3a^2b^2(2b + 3a)$

Insert parentheses for the expression.

$6ab^2 + 9a^2b$       Write the expression.

$3ab \cdot 2b + 3ab \cdot 3a$       Identify a common factor.

$3ab(2b + 3a)$       Distributive Property

The expression  $6ab^2 + 9a^2b$  is equivalent to  $3ab(2b + 3a)$ .  
The correct answer is choice C.

60. Which of the following is equivalent to the expression  $(5m - 3)12$ ?

TEKS A.10(D) MP A.1(D), A.1(F)

- A  $5m - 36$
- B  $17m + 9$
- C  $60m - 36$
- D  $60m + 36$

61. Which of the following is equivalent to the expression  $3m - 7m^2 + 5m + m^2$ ? TEKS A.10(D)

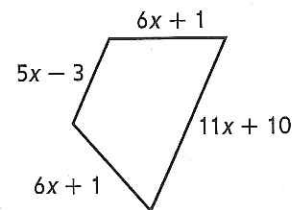
MP A.1(D), A.1(F)

- I.  $-6m^2 + 8m$
- II.  $2m(4m - 3)$
- III.  $2m(4 - 3m)$

- F I only
- G I and II
- H I and III
- J III only

62. **GRIDDABLE** If  $2x(3 + 4a) = -16x + 6x$ , what is the value of  $a$ ? TEKS A.10(D) MP A.1(B), A.1(D)

63. The diagram shows the side lengths of a quadrilateral.



Which expression represents the perimeter of the figure? TEKS A.10(D) MP A.1(D), A.1(E)

- A  $22x + 8$
- B  $27x + 9$
- C  $28x + 9$
- D  $9(3x + 1)$

64. What is the simplified form of the expression  $3y^2 + 7 - y^2 - 5 - 2y^2 - 1$ ? TEKS A.10(D)

MP A.1(D), A.1(F)

- F 0
- G 1
- H 2
- J  $6y^2 + 1$

## Mid-Chapter Quiz

Lessons 1-1 through 1-4

Write a verbal expression for each algebraic expression.

(Lesson 1-1)

1.  $21 - x^3$                       2.  $3m^5 + 9$

Write an algebraic expression for each verbal expression. (Lesson 1-1)

3. five more than  $s$  squared  
 4. four times  $y$  to the fourth power  
 5. **CAR RENTAL** The XYZ Car Rental Agency charges a flat rate of \$39 per day plus \$0.47 per mile driven. Write an algebraic expression for the rental cost of a car for  $x$  days that is driven  $y$  miles. (Lesson 1-1)

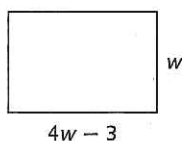
Evaluate each expression. (Lesson 1-2)

6.  $24 \div 3 - 2 \cdot 3$   
 7.  $5 + 2^2$   
 8.  $4(3 + 9)$   
 9.  $36 - 2(1 + 3)^2$   
 10.  $\frac{40 - 2^3}{4 + 3(2^2)}$

11. **AMUSEMENT PARK** The costs of tickets to a local amusement park are shown. Write and evaluate an expression to find the total cost for 5 adults and 8 children. (Lesson 1-2)



12. **MULTIPLE CHOICE** Write an algebraic expression to represent the perimeter of the rectangle shown below. Then evaluate it to find the perimeter when  $w = 8$  cm. (Lesson 1-2)



- A 37 cm                      C 74 cm  
 B 232 cm                    D 45 cm

Evaluate each expression. Name the property used in each step. (Lesson 1-3)

13.  $(8 - 2^3) + 21$   
 14.  $3(1 \div 3) \cdot 9$   
 15.  $[5 \div (3 \cdot 1)]^{\frac{3}{5}}$   
 16.  $18 + 35 + 32 + 15$   
 17.  $0.25 \cdot 7 \cdot 4$

Use the Distributive Property to rewrite each expression. Then evaluate. (Lesson 1-4)

18.  $3(5 + 2)$   
 19.  $(9 - 6)12$   
 20.  $8(7 - 4)$

Use the Distributive Property to rewrite each expression. Then simplify. (Lesson 1-4)

21.  $4(x + 3)$   
 22.  $(6 - 2y)7$   
 23.  $-5(3m - 2)$

24. **CAR WASH** A car wash chain has three locations. Use the information in the table below to write and evaluate an expression to estimate the total number of car washes sold over a 4-day period. (Lesson 1-4)

Location	Daily Car Washes
Location 1	145
Location 2	211
Location 3	184

25. **MULTIPLE CHOICE** Rewrite the expression  $(8 - 3p)(-2)$  using the Distributive Property. (Lesson 1-4)

- F  $16 - 6p$   
 G  $-10p$   
 H  $-16 + 6p$   
 J  $10p$

# 1-5 Equations



**Then**

You simplified expressions.

**Now**

- Solve equations with one variable.
- Solve equations with two variables.

**Why?**

Mark's baseball team scored 3 runs in the first inning. At the end of the second inning, their score was 4. The open sentence below represents the change in their score.

$$3 + r = 4$$

The solution is 1. The team got 1 run in the second inning.



**Targeted TEKS**

**A.5(A)** Solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

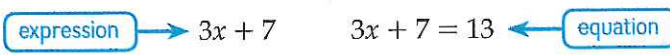


**Mathematical Processes**

**A.1(B)** Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

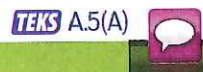
**A.1(E)** Create and use representations to organize, record, and communicate mathematical ideas.

**1 Solve Equations** A mathematical statement that contains algebraic expressions and symbols is an **open sentence**. A sentence that contains an equals sign, =, is an **equation**.



Finding a value for a variable that makes a sentence true is called **solving** the open sentence. This replacement value is a **solution**.

A set of numbers from which replacements for a variable may be chosen is called a **replacement set**. A **set** is a collection of objects or numbers that is often shown using braces. Each object or number in the set is called an **element**, or member. A **solution set** is the set of elements from the replacement set that make an open sentence true.



**Example 1 Use a Replacement Set**

Find the solution set of the equation  $2q + 5 = 13$  if the replacement set is  $\{2, 3, 4, 5, 6\}$ .

Use a table to solve. Replace  $q$  in  $2q + 5 = 13$  with each value in the replacement set.

$q$	$2q + 5 = 13$	True or False?
2	$2(2) + 5 = 13$	false
3	$2(3) + 5 = 13$	false
4	$2(4) + 5 = 13$	true
5	$2(5) + 5 = 13$	false
6	$2(6) + 5 = 13$	false

Since the equation is true when  $q = 4$ , the solution of  $2q + 5 = 13$  is  $q = 4$ .

The solution set is  $\{4\}$ .

**Guided Practice**

Find the solution set for each equation if the replacement set is  $\{0, 1, 2, 3\}$ .

**1A.**  $8m - 7 = 17$

**1B.**  $28 = 4(1 + 3d)$



**New Vocabulary**

- open sentence
- equation
- solving
- solution
- replacement set
- set
- element
- solution set
- identity



You can often solve an equation by applying the order of operations.

TEKS A.5(A)



### Example 2 Apply the Order of Operations

Solve  $6 + (5^2 - 5) \div 2 = p$ .

- A 3                      B 6                      C 13                      D 16

#### Read the Item

You need to apply the order of operations to the expression in order to solve for  $p$ .

#### Solve the Item

$$6 + (5^2 - 5) \div 2 = p \quad \text{Original equation}$$

$$6 + (25 - 5) \div 2 = p \quad \text{Evaluate powers.}$$

$$6 + 20 \div 2 = p \quad \text{Subtract 5 from 25.}$$

$$6 + 10 = p \quad \text{Divide 20 by 2.}$$

$$16 = p \quad \text{Add.}$$

The correct answer is D.

#### Guided Practice

2. Solve  $t = 9^2 \div (5 - 2)$ .

- F 3                      G 6                      H 14.2                      J 27

#### StudyTip

##### Rewrite the Equation

If you are allowed to write in your testing booklet, it can be helpful to rewrite the equation with simplified terms.

Some equations have a unique solution. Other equations do not have a solution.

TEKS A.5(A)



### Example 3 Solutions of Equations

Solve each equation.

a.  $7 - (4^2 - 10) + n = 10$

Simplify the equation first and then look for a solution.

$$7 - (4^2 - 10) + n = 10 \quad \text{Original equation}$$

$$7 - (16 - 10) + n = 10 \quad \text{Evaluate powers.}$$

$$7 - 6 + n = 10 \quad \text{Subtract 10 from 16.}$$

$$1 + n = 10 \quad \text{Subtract 6 from 7.}$$

The only value for  $n$  that makes the equation true is 9. Therefore, this equation has a unique solution of 9.

b.  $n(3 + 2) + 6 = 5n + (10 - 3)$

$$n(3 + 2) + 6 = 5n + (10 - 3) \quad \text{Original equation}$$

$$n(5) + 6 = 5n + (10 - 3) \quad \text{Add 3 + 2.}$$

$$n(5) + 6 = 5n + 7 \quad \text{Subtract 3 from 10.}$$

$$5n + 6 = 5n + 7 \quad \text{Commutative (}\times\text{)}$$

No matter what real value is substituted for  $n$ , the left side of the equation will always be one less than the right side. So, the equation will never be true. Therefore, there is no solution of this equation.

#### Guided Practice

3A.  $(18 + 4) + m = (5 - 3)m$

3B.  $8 \cdot 4 \cdot k + 9 \cdot 5 = (36 - 4)k - (2 \cdot 5)$

#### StudyTip

**Guess and Check** When the solution to an equation is not easy to see, substitute values for  $x$  and test the equation. Continue to test values until you get a true statement. For example, if  $3x + 16 = 73$ , test values for  $x$ .

$$3(10) + 16 = 48 \quad \text{too low}$$

$$3(20) + 16 = 76 \quad \text{too high}$$

$$3(19) + 16 = 73 \quad \checkmark$$

## ReadingMath

**Identities** You may know twins who are *identical*, meaning that genetically they are *exactly the same*. You can remember the mathematical meaning of *identity* as an equation that shows that a number or expression is *identical* to itself.

An equation that is true for every value of the variable is called an **identity**.

TEKS A.5(A)



### Example 4 Identities

$$\text{Solve } (2 \cdot 5 - 8)(3h + 6) = [(2h + h) + 6]2.$$

$$(2 \cdot 5 - 8)(3h + 6) = [(2h + h) + 6]2 \quad \text{Original Equation}$$

$$(10 - 8)(3h + 6) = [(2h + h) + 6]2 \quad \text{Multiply } 2 \cdot 5.$$

$$2(3h + 6) = [(2h + h) + 6]2 \quad \text{Subtract 8 from 10.}$$

$$6h + 12 = [(2h + h) + 6]2 \quad \text{Distributive Property}$$

$$6h + 12 = [3h + 6]2 \quad \text{Add } 2h + h.$$

$$6h + 12 = 6h + 12 \quad \text{Distributive Property}$$

No matter what value is substituted for  $h$ , the left side of the equation will always be equal to the right side. So, the equation will always be true. Therefore, the solution of this equation could be any real number.

### Guided Practice

Solve each equation.

4A.  $12(10 - 7) + 9g = g(2^2 + 5) + 36$

4B.  $2d + (2^3 - 5) = 10(5 - 2) + d(12 \div 6)$

4C.  $3(b + 1) - 5 = 3b - 2$

4D.  $5 - \frac{1}{2}(c - 6) = 4$

## Go Online!

Got a question? Send a message to your teacher in ConnectED.

**2 Solve Equations with Two Variables** Some equations contain two variables. It is often useful to make a table of values and use substitution to find the corresponding values of the second variable.

TEKS A.5(A)



### Example 5 Equations Involving Two Variables

**PADDLE BOARD** Juanita pays \$15 per hour to rent a paddle board. She can rent a wet suit for \$5 a day. Write and solve an equation to find the total amount Juanita spends in one day if she boards 4 hours and rents a wet suit.

The rental for the wet suit is a flat rate. The variable is the number of hours she rents the board. The total cost is the price of the wet suit plus \$15 times the number of hours she paddle boards. Let  $C$  be the total cost and  $h$  be the number of hours she spends with the board.

$$C = 15h + 5 \quad \text{Original equation}$$

$$= 15(4) + 5 \quad \text{Substitute 4 for } h.$$

$$= 60 + 5 \quad \text{Multiply.}$$

$$= 65 \quad \text{Simplify.}$$

Juanita will spend \$65 for 4 hours of paddle boarding.

### Guided Practice

5. **TRAVEL** Amelia drives an average of 65 miles per hour. Write and solve an equation to find the time it will take her to drive 36 miles.

**Example 1** Find the solution set of each equation if the replacement set is {11, 12, 13, 14, 15}.**TEKS** A.5(A)

1.  $n + 10 = 23$

2.  $7 = \frac{c}{2}$

3.  $29 = 3x - 7$

4.  $(k - 8)12 = 84$

**Example 2** Solve each equation.**TEKS** A.5(A)

5.  $\frac{d + 5}{10} = 2$

6.  $\frac{2}{3}b + 4 = 6$

7.  $23 - 7y = 3^2$

**Examples 3–4** Solve each equation.**TEKS** A.5(A)

8.  $x = 4(6) + 3$

9.  $14 - 82 = w$

10.  $5 + 22a = 2 + 10 \div 2$

11.  $(2 \cdot 5) + \frac{c^3}{3} = c^3 \div (1^5 + 2) + 10$

**Example 5****TEKS** A.5(A)

12. **RECYCLING** Round Rock, Texas, has a recycling facility that accepts unused paint. Volunteers blend and mix the paint and give it away in 5-gallon buckets. Write and solve an equation to find the number of buckets of paint given away from the 30,000 gallons that are donated.

## Practice and Problem Solving

Extra Practice is on page R1.

**Example 1****TEKS** A.5(A)

Find the solution set of each equation if the replacement sets are  $y$ : {1, 3, 5, 7, 9} and  $z$ : {10, 12, 14, 16, 18}.

13.  $z + 10 = 22$

14.  $52 = 4z$

15.  $\frac{15}{y} = 3$

16.  $17 = 24 - y$

17.  $2z - 5 = 27$

18.  $4(y + 1) = 40$

19.  $22 = \frac{60}{y} + 2$

20.  $111 = z^2 + 11$

**Examples 2–4** Solve each equation.**TEKS** A.5(A)

21.  $a = 32 - 9(2)$

22.  $w = 56 \div (2^2 + 3)$

23.  $\frac{27 + 5}{16} = g$

24.  $\frac{12 \cdot 5}{15 - 3} = y$

25.  $r = \frac{9(6)}{(8 + 1)3}$

26.  $a = \frac{4(14 - 1)}{3(6) - 5} + 7$

27.  $(4 - 2^2 + 5)w = 25$

28.  $7 + x - (3 + 32 \div 8) = 3$

29.  $3^2 - 2 \cdot 3 + u = (3^3 - 3 \cdot 8)(2) + u$

30.  $(3 \cdot 6 \div 2)v + 10 = 3^2v + 9$

31.  $6k + (3 \cdot 10 - 8) = (2 \cdot 3)k + 22$

32.  $(3 \cdot 5)t + (21 - 12) = 15t + 3^2$

33.  $(2^4 - 3 \cdot 5)q + 13 = (2 \cdot 9 - 4^2)q + \left(\frac{3 \cdot 4}{12} - 1\right)$

34.  $\frac{3 \cdot 22}{18 + 4}r - \left(\frac{4^2}{9 + 7} - 1\right) = r + \left(\frac{8 \cdot 9}{3} \div 3\right)$

35. **SCHOOL** A conference room can seat a maximum of 85 people. The principal and two counselors need to meet with the school's juniors to discuss college admissions. If each student must bring a parent with them, how many students can attend each meeting? Assume that each student has a unique set of parents.

36. **MP PROBLEM SOLVING** The perimeter of a regular octagon is 128 inches. Find the length of each side.

**Example 5**  
**TEKS** A.5(A)

**37. SPORTS** A 200-pound athlete who trains for four hours per day requires 2836 Calories for basic energy requirements. During training, the same athlete requires an additional 3091 Calories for extra energy requirements. Write an equation to find  $C$ , the total daily Calorie requirement for this athlete. Then solve the equation.

**38. ENERGY** An electric generator can power 3550 watts of electricity. Write and solve an equation to find how many 75-watt light bulbs a generator could power.

Make a table of values for each equation if the replacement set is  $\{-2, -1, 0, 1, 2\}$ .

**39.**  $y = 3x - 2$

**40.**  $3.25x + 0.75 = y$

Solve each equation using the given replacement set.

**41.**  $t - 13 = 7$ ;  $\{10, 13, 17, 20\}$

**42.**  $14(x + 5) = 126$ ;  $\{3, 4, 5, 6, 7\}$

**43.**  $22 = \frac{n}{3}$ ;  $\{62, 64, 66, 68, 70\}$

**44.**  $35 = \frac{g - 8}{2}$ ;  $\{78, 79, 80, 81\}$

Solve each equation.

**45.**  $\frac{3(9) - 2}{1 + 4} = d$

**46.**  $j = 15 \div 3 \cdot 5 - 4^2$

**47.**  $c + (3^2 - 3) = 21$

**48.**  $(3^3 - 3 \cdot 9) + (7 - 2^2)b = 24b$

**49. MP ORGANIZE IDEAS** Blood flow rate can be expressed as  $F = \frac{p_1 - p_2}{r}$ , where  $F$  is the flow rate,  $p_1$  and  $p_2$  are the initial and final pressure exerted against the blood vessel's walls, respectively, and  $r$  is the resistance created by the size of the vessel.

- Write and solve an equation to determine the resistance of the blood vessel for an initial pressure of 100 millimeters of mercury, a final pressure of 0 millimeters of mercury, and a flow rate of 5 liters per minute.
- Use the equation to complete the table below.

Initial Pressure $p_1$ (mm Hg)	Final Pressure $p_2$ (mm Hg)	Resistance $r$ (mm Hg/L/min)	Blood Flow Rate $F$ (L/min)
100	0		5
100	0	30	
	5	40	4
90		10	6

Determine whether the given number is a solution of the equation.

**50.**  $x + 6 = 15$ ; 9

**51.**  $12 + y = 26$ ; 14

**52.**  $2t - 10 = 4$ ; 3

**53.**  $3r + 7 = -5$ ; 2

**54.**  $6 + 4m = 18$ ; 3

**55.**  $-5 + 2p = -11$ ; -3

**56.**  $\frac{q}{2} = 20$ ; 10

**57.**  $\frac{w - 4}{5} = -3$ ; -11

**58.**  $\frac{g}{3} - 4 = 12$ ; 48

Make a table of values for each equation if the replacement set is  $\{-2, -1, 0, 1, 2\}$ .

**59.**  $y = 3x + 5$

**60.**  $-2x - 3 = y$

**61.**  $y = \frac{1}{2}x + 2$

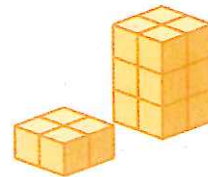
**62.**  $4.2x - 1.6 = y$

**63. GEOMETRY** The length of a rectangle is 2 inches greater than the width. The length of the base of an isosceles triangle is 12 inches, and the lengths of the other two sides are 1 inch greater than the width of the rectangle.

- Draw a picture of each figure and label the dimensions.
- Write two expressions to find the perimeters of the rectangle and triangle.
- Find the width of the rectangle if the perimeters of the figures are equal.

64. **MULTI-STEP** A company is moving into a new office which has 32 feet by 150 feet of space reserved for cubicles. Individual cubicles require 8 feet by 10 feet of space. Pod cubicles, which are not preferred by employees, can hold up to 4 people and require 10 feet by 16 feet of space. There is 5 feet of space between each row of cubicles. The company currently has 46 employees designated for cubicles.
- How many pods will they need?
  - Describe your solution process.
  - What assumptions did you make?

65. **MP MULTIPLE REPRESENTATIONS** In this problem, you will further explore writing equations.



- Concrete** Use centimeter cubes to build a tower similar to the one shown at the right.
- Tabular** Copy and complete the table shown below. Record the number of layers in the tower and the number of cubes used in the table.

Layers	1	2	3	4	5	6	7
Cubes	?	?	?	?	?	?	?

- Analytical** As the number of layers in the tower increases, how does the number of cubes in the tower change?
- Algebraic** Write a rule that gives the number of cubes in terms of the number of layers in the tower.

TEKS A.5(A)

### H.O.T. Problems Use Higher-Order Thinking Skills

- MP ANALYZE RELATIONSHIPS** Compare and contrast an expression and an equation.
- MP ORGANIZE IDEAS** Write an equation that is an identity.
- MP ANALYZE RELATIONSHIPS** Explain why an open sentence always has at least one variable.
- ERROR ANALYSIS** Tom and Li-Cheng are solving the equation  $x = 4(3 - 2) + 6 \div 8$ . Is either of them correct? Explain your reasoning.

<p><b>Tom</b></p> $  \begin{aligned}  x &= 4(3 - 2) + 6 \div 8 \\  &= 4(1) + 6 \div 8 \\  &= 4 + 6 \div 8 \\  &= 4 + \frac{6}{8} \\  &= 4 \frac{3}{4}  \end{aligned}  $	<p><b>Li-Cheng</b></p> $  \begin{aligned}  x &= 4(3 - 2) + 6 \div 8 \\  &= 4(1) + 6 \div 8 \\  &= 4 + 6 \div 8 \\  &= 10 \div 8 \\  &= \frac{5}{4}  \end{aligned}  $
---	--

- MP PROBLEM SOLVING** Find all of the solutions of  $x^2 + 5 = 30$ .
- MP ORGANIZE IDEAS** Write an equation that involves two or more operations with a solution of  $-7$ .
- WRITING IN MATH** Explain how you can determine that an equation has no real numbers as a solution. How can you determine that an equation has all real numbers as solutions?



TEKS A.5(A) MP A.1(C), A.1(D)

## Example

What is the solution to the equation  $\frac{p^2 + 5}{2} = 27$ ?

- A 0
- B 6
- C 7
- D 10

Substitute each value into the equation.

If  $p = 0$ , then  $\frac{0^2 + 5}{2} = \frac{5}{2} = 2.5$ , and  $2.5 \neq 27$ .

If  $p = 6$ , then  $\frac{6^2 + 5}{2} = \frac{41}{2} = 20.5$ , and  $20.5 \neq 27$ .

If  $p = 7$ , then  $\frac{7^2 + 5}{2} = \frac{54}{2} = 27$ .

If  $p = 10$ , then  $\frac{10^2 + 5}{2} = \frac{105}{2} = 52.5$ , and  $52.5 \neq 27$ .

The value  $p = 7$  satisfies the equation. The correct answer is choice C.

73. What is the solution to the equation  $t = 5 - (3^2 + 1) + 2$ ? **TEKS** A.5(A) **MP** A.1(C), A.1(D)

- A -3
- B -1
- C 0
- D 17

74. Use the order of operations to simplify both sides of  $m(15 + 1) + 7 = 11m + (15 - 9) + 5m$ . What is the solution?

**TEKS** A.5(A), **MP** A.1(C), A.1(G)

- F  $16m + 7 = 16m + 6$ ; all real numbers
- G  $16m + 7 = 16m + 6$ ; no solution
- H  $16m + 7 = 16m + 7$ ; all real numbers
- J  $16m + 7 = 16m + 7$ ; no solution

75. What is the solution to the equation  $5(x - 7) = 20$ ? **TEKS** A.5(A) **MP** A.1(C), A.1(D)

- A 3
- B 7
- C 8
- D 11

76. A large container of juice costs \$1.95 less than 3 times the cost of a small container of juice. What is the cost of a large container of juice if the cost of a small container is \$1.60?

**TEKS** A.5(A) **MP** A.1(A), A.1(C)

- F \$1.60
- G \$2.85
- H \$2.92
- J \$3.55

77. The total resistance  $T$  in ohms of two resistors with resistance  $R_1$  and  $R_2$  is given by the equation:

$$\frac{1}{T} = \frac{1}{R_1} + \frac{1}{R_2}$$

If the total resistance is 12 ohms and one resistor has a resistance of 30 ohms, what is the resistance of the other resistor? **TEKS** A.5(A)

**MP** A.1(A), A.1(F)

- A  $\frac{1}{12}$  ohm
- B 2.5 ohms
- C 20 ohms
- D 42 ohms

# LESSON 1-6 Relations

Then

- You solved equations with one or two variables.

Now

- 1 Represent relations.
- 2 Interpret graphs of relations.

Why?

- The deeper in the ocean you are, the greater pressure is on your body. This is because there is more water over you. The force of gravity pulls the water weight down, creating a greater pressure.

The equation that relates the total pressure of the water to the depth is  $P = rgh$ , where

- $P$  = the pressure,
- $r$  = the density of water,
- $g$  = the acceleration due to gravity, and
- $h$  = the height of water above you.



### Targeted TEKS

**A.2(A)** Determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete, and represent domain and range using inequalities.



### Mathematical Processes

**A.1(A)** Apply mathematics to problems arising in everyday life, society, and the workplace.

**A.1(E)** Create and use representations to organize, record, and communicate mathematical ideas.



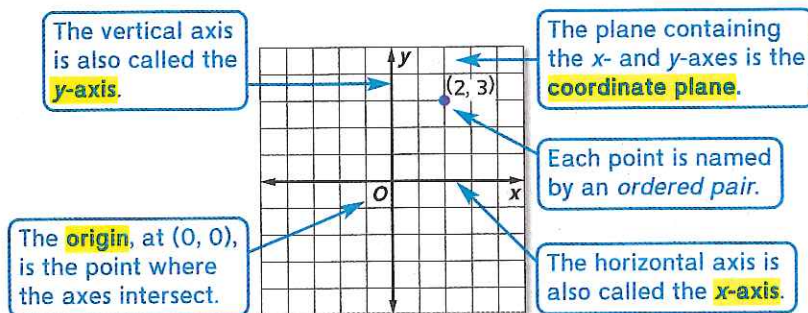
### New Vocabulary

coordinate system  
coordinate plane  
 $x$ - and  $y$ -axes  
origin  
ordered pair  
 $x$ - and  $y$ -coordinates  
relation  
mapping  
domain  
range  
independent variable  
dependent variable

## 1 Represent a Relation

This relationship between the depth and the pressure exerted can be represented by a line on a coordinate grid.

A **coordinate system** is formed by the intersection of two number lines, the *horizontal axis* and the *vertical axis*.

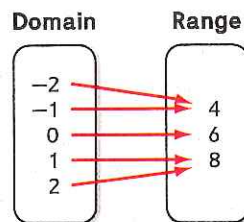


A point is represented on a graph using ordered pairs.

- An **ordered pair** is a set of numbers, or *coordinates*, written in the form  $(x, y)$ .
- The  $x$ -value, called the  **$x$ -coordinate**, represents the horizontal placement of the point.
- The  $y$ -value, or  **$y$ -coordinate**, represents the vertical placement of the point.

A set of ordered pairs is called a **relation**. A relation can be represented in several different ways: as an equation, in a graph, with a table, or with a mapping.

A **mapping** illustrates how each element of the *domain* is paired with an element in the *range*. The set of the first numbers of the ordered pairs is the **domain**. The set of second numbers of the ordered pairs is the **range** of the relation. This mapping represents the ordered pairs  $(-2, 4)$ ,  $(-1, 4)$ ,  $(0, 6)$ ,  $(1, 8)$ , and  $(2, 8)$ .



### StudyTip

**MP Organize Ideas** Each representation of the same relation serves a different purpose. Graphing the points can show the pattern between the points. A mapping shows you at a glance the pairings between elements in the domain and range.

Study the different representations of the same relation below.

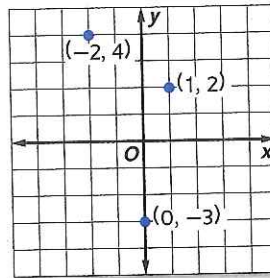
#### Ordered Pairs

$(1, 2)$   
 $(-2, 4)$   
 $(0, -3)$

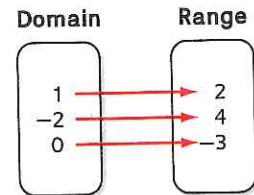
#### Table

x	y
1	2
-2	4
0	-3

#### Graph



#### Mapping



The  $x$ -values of a relation are members of the domain and the  $y$ -values of a relation are members of the range. In the relation above, the domain is  $\{-2, 1, 0\}$  and the range is  $\{-3, 2, 4\}$ .

TEKS A.2(A)



### Go Online!

Log into your **eStudent Edition** to hear pronunciations of vocabulary terms or to listen to complete audio of the text. An online dictionary may be useful for basic terms. Ask for clarification as you need it.

### Example 1 Representations of a Relation

a. Express  $\{(2, 5), (-2, 3), (5, -2), (-1, -2)\}$  as a table, a graph, and a mapping.

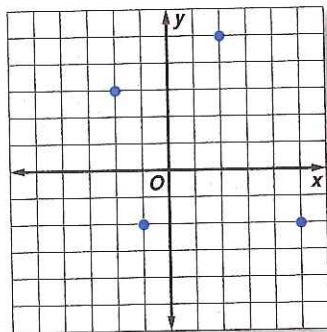
#### Table

Place the  $x$ -coordinates into the first column of the table. Place the corresponding  $y$ -coordinates in the second column of the table.

x	y
2	5
-2	3
5	-2
-1	-2

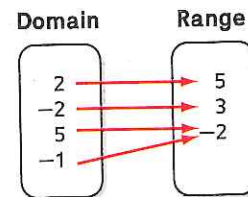
#### Graph

Graph each ordered pair on a coordinate plane.



#### Mapping

List the  $x$ -values in the domain and the  $y$ -values in the range. Draw arrows from the  $x$ -values in the domain to the corresponding  $y$ -values in the range.



b. Determine the domain and the range of the relation.

The domain of the relation is  $\{2, -2, 5, -1\}$ . The range of the relation is  $\{5, 3, -2\}$ .

### GuidedPractice

1A. Express  $\{(4, -3), (3, 2), (-4, 1), (0, -3)\}$  as a table, graph, and mapping.

1B. Determine the domain and range.



In a relation, the value of the variable that determines the output is called the **independent variable**. The variable with a value that is dependent on the value of the independent variable is called the **dependent variable**. The domain contains values of the independent variable. The range contains the values of the dependent variable.

### Real-World Example 2 Independent and Dependent Variables

Identify the independent and dependent variables for each relation.

- a. **DANCE** The dance committee is selling tickets to the Fall Ball. The more tickets that they sell, the greater the amount of money they can spend for decorations.

The number of tickets sold is the independent variable because it is unaffected by the money spent on decorations. The money spent on decorations is the dependent variable because it depends on the number of tickets sold.

- b. **MOVIES** Generally, the average price of going to the movies has steadily increased over time.

Time is the independent variable because it is unaffected by the cost of attending the movies. The price of going to the movies is the dependent variable because it is affected by time.

#### Guided Practice

Identify the independent and dependent variables for each relation.

- 2A. The air pressure inside a tire increases with the temperature.  
2B. As the amount of rain decreases, so does the water level of the river.



#### Real-WorldLink

In 1948, a movie ticket cost \$0.36. In 2011, the average ticket price in the United States was \$7.93.

Source: National Association of Theatre Owners

## 2 Graphs of a Relation

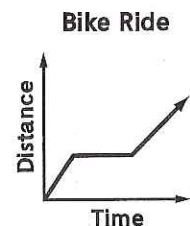
A relation can be graphed without a scale on either axis. These graphs can be interpreted by analyzing their shape.

### Example 3 Analyze Graphs

The graph represents the distance Francesca has ridden on her bike. Describe what happens in the graph.

As time increases, the distance increases until the graph becomes a horizontal line.

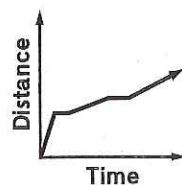
So, time is increasing but the distance remains constant. At this section Francesca stopped. Then she continued to ride her bike.



#### Guided Practice

Describe what is happening in each graph.

- 3A. Driving to School



- 3B. Change in Income



**Example 1**  
**TEKS** A.2(A)

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1.  $\{(4, 3), (-2, 2), (5, -6)\}$                       2.  $\{(5, -7), (-1, 4), (0, -5), (-2, 3)\}$

**Example 2**

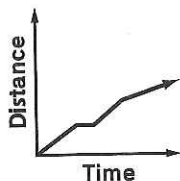
Identify the independent and dependent variables for each relation.

- Increasing the temperature of a compound inside a sealed container increases the pressure inside a sealed container.
- Mike's cell phone is part of a family plan. If he uses more minutes than his share, then there are fewer minutes available for the rest of his family.
- Julian is buying concert tickets for himself and his friends. The more concert tickets he buys the greater the cost.
- A store is having a sale over Labor Day weekend. The more purchases, the greater the profits.

**Example 3**

**MP APPLY MATH** Describe what is happening in each graph.

- The graph represents the distance the track team runs during a practice.
- The graph represents revenues generated through an online store.



Practice and Problem Solving

Extra Practice is on page R1.

**Example 1**  
**TEKS** A.2(A)

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

- $\{(0, 0), (-3, 2), (6, 4), (-1, 1)\}$
- $\{(5, 2), (5, 6), (3, -2), (0, -2)\}$
- $\{(6, 1), (4, -3), (3, 2), (-1, -3)\}$
- $\{(-1, 3), (3, -6), (-1, -8), (-3, -7)\}$
- $\{(6, 7), (3, -2), (8, 8), (-6, 2), (2, -6)\}$
- $\{(4, -3), (1, 3), (7, -2), (2, -2), (1, 5)\}$

**Example 2**

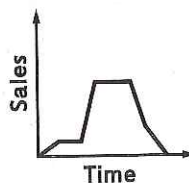
Identify the independent and dependent variables for each relation.

- The Spanish classes are having a fiesta lunch. Each student that attends is to bring a Spanish side dish or dessert. The more students that attend, the more food there will be available.
- The faster you drive your car, the longer it will take to come to a complete stop.

**Example 3**

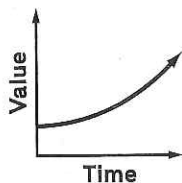
**MP APPLY MATH** Describe what is happening in each graph.

- The graph represents the height of a bungee jumper.
- The graph represents the sales of lawn mowers.

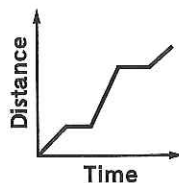


**MP APPLY MATH** Describe what is happening in each graph.

19. The graph represents the value of a rare baseball card.

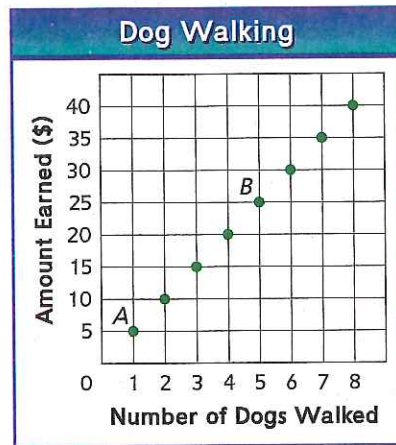


20. The graph represents the distance covered on an extended car ride.



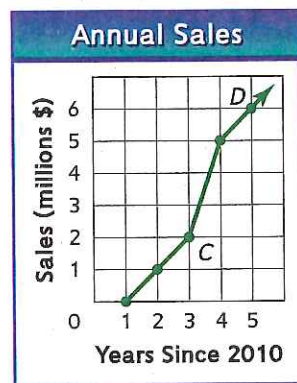
For Exercises 21–23, use the graph at the right.

21. Name the ordered pair at point *A* and explain what it represents.
22. Name the ordered pair at point *B* and explain what it represents.
23. Identify the independent and dependent variables for the relation.



For Exercises 24–26, use the graph at the right.

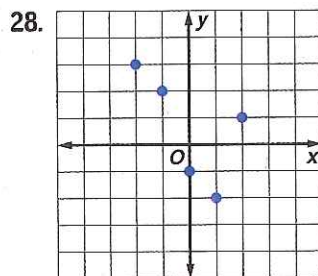
24. Name the ordered pair at point *C* and explain what it represents.
25. Name the ordered pair at point *D* and explain what it represents.
26. Identify the independent and dependent variables.



Express each relation as a set of ordered pairs. Describe the domain and range.

27. **Buying Aquarium Fish**

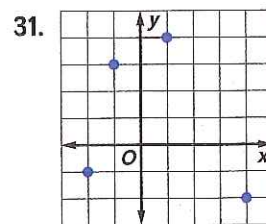
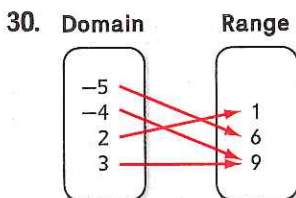
Number of Fish	Total Cost
1	\$2.50
2	\$4.50
5	\$10.50
8	\$16.50



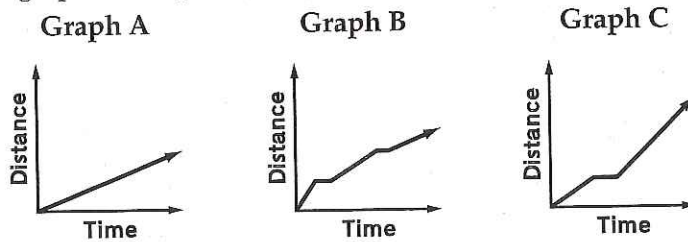
Express the relation in each table, mapping, or graph as a set of ordered pairs.

29.

<i>x</i>	<i>y</i>
4	-1
8	9
-2	-6
7	-3



32. **SPORTS** In a triathlon, athletes swim 2.4 miles, bicycle 112 miles, and run 26.2 miles. Their total time includes transition time from one activity to the next. Which graph best represents a participant in a triathlon? Explain.



Draw a graph to represent each situation.

33. **ANTIQUES** A grandfather clock that is over 100 years old has increased in value from when it was first purchased.
34. **CAR** A car depreciates in value. The value decreases quickly in the first few years.
35. **REAL ESTATE** A house typically increases in value over time.
36. **EXERCISE** An athlete alternates between running and walking during a workout.
37. **PHYSIOLOGY** A typical adult has about 2 pounds of water for every 3 pounds of body weight. This can be represented by the equation  $w = 2\left(\frac{b}{3}\right)$ , where  $w$  is the weight of water in pounds and  $b$  is the body weight in pounds.
- Make a table to show the relation between body and water weight for people weighing 100, 105, 110, 115, 120, 125, and 130 pounds. Round to the nearest tenth if necessary.
  - What are the independent and dependent variables?
  - State the domain and range, and then graph the relation.
  - Reverse the independent and dependent variables. Graph this relation. Explain what the graph indicates in this circumstance.

TEKS A.2(A)

**H.O.T. Problems** Use Higher-Order Thinking Skills

38. **MP ORGANIZE IDEAS** Describe a real-life situation that can be represented using a relation and discuss how one of the quantities in the relation depends on the other. Then represent the relation in three different ways.
39. **MP ANALYZE RELATIONSHIPS** Describe a real-world situation where it is reasonable to have a negative number included in the domain or range.
40. **MP APPLY MATH** Compare and contrast dependent and independent variables.
41. **MP PROBLEM SOLVING** The table presents a relation. Graph the ordered pairs. Then reverse the  $y$ -coordinate and the  $x$ -coordinate in each ordered pair. Graph these ordered pairs on the same coordinate plane. Graph the line  $y = x$ . Describe the relationship between the two sets of ordered pairs.
42. **WRITING IN MATH** Use the data about the pressure of water on page 40 to explain the difference between dependent and independent variables.

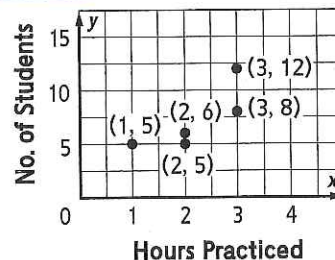
$x$	$y$
0	1
1	3
2	5
3	7



## Example

TEKS A.2(A) MP A.1(A), A.1(D)

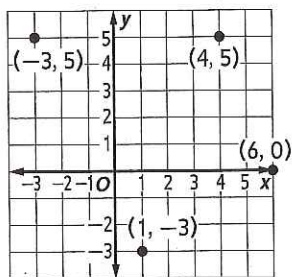
The trumpet section of the marching band was asked how many hours they spent practicing for the homecoming game. Their responses were recorded in the graph shown. Which best describes the relation (number of trumpet players, number of hours practiced)?



- A domain: {1, 2, 2, 3, 3}; range: {5, 6, 8, 12}
- B domain: {1, 2, 3}; range: {5, 6, 8, 12}
- C domain: {5, 6, 8, 12}; range: {1, 2, 3}
- D domain: {1, 2, 3}; range: {4, 5, 6}

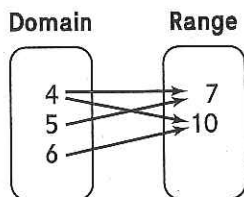
The domain for a set of ordered pairs is a list of the first elements in the ordered pairs, with no duplicates. So the domain is {1, 2, 3}. The range is the list of the second elements in the ordered pairs, with no duplicates. So the range is {5, 6, 8, 12}. The correct answer is choice B.

43. The points on this graph represent a relation. What are the domain and range of the relation? **TEKS** A.2(A) **MP** A.1(D)



- A domain: {-3, 0, 5}; range: {-3, 1, 4, 6}
- B domain: {-3, 1, 1, 4, 6}; range: {-3, 0, 5, 5}
- C domain: {-3, 1, 4, 6}; range: {-3, 0, 5, 5}
- D domain: {-3, 1, 4, 6}; range: {-3, 0, 5}

44. A mapping is shown.



Which set of ordered pairs represents the same relation as the mapping? **TEKS** A.2(A) **MP** A.1(D), A.1(E)

- F {(7, 4), (7, 5), (10, 4), (10, 6)}
- G {(4, 7), (5, 7), (6, 10)}
- H {(4, 7), (4, 10), (5, 7), (6, 10)}
- J {(4, 7), (4, 10), (5, 7), (6, 7), (6, 10)}

45. Three of the representations below show the same relation. Which answer choice shows a relation that is different from the other three? **TEKS** A.2(A) **MP** A.1(D), A.1(F)

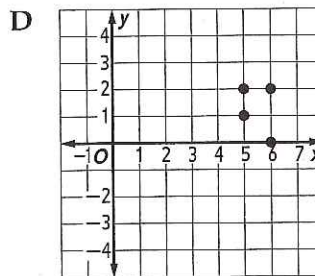
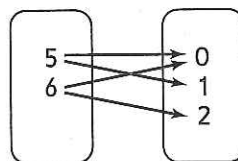
**TEKS** A.2(A) **MP** A.1(D), A.1(F)

A

<i>x</i>	5	5	6	6
<i>y</i>	0	1	0	2

- B {(5, 0), (5, 1), (6, 0), (6, 2)}

C Domain Range



**Then**

- You solved equations with elements from a replacement set.

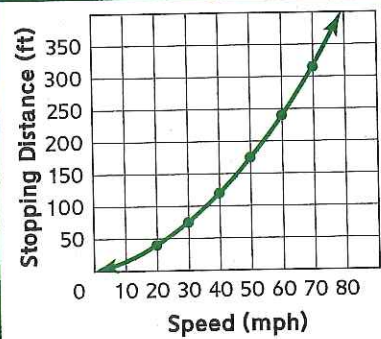
**Now**

- Determine whether a relation is a function.
- Find function values.

**Why?**

- The distance a car travels from when the brakes are applied to the car's complete stop is the stopping distance. This includes time for the driver to react. The faster a car is traveling, the longer the stopping distance. The stopping distance is a function of the speed of the car.

Stopping Distance of a Passenger Car



**Targeted TEKS**

**A.12(A)** Decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

**A.12(B)** Evaluate functions, expressed in function notation, given one or more elements in their domains.

Also addresses A.2(A).



**Mathematical Processes**

**A.1(B)** Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

**A.1(E)** Create and use representations to organize, record, and communicate mathematical ideas.



**New Vocabulary**

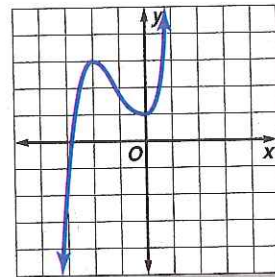
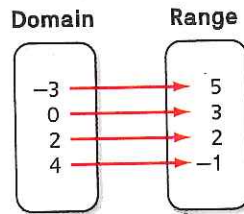
- function
- discrete function
- continuous function
- vertical line test
- function notation
- nonlinear function

**1 Identify Functions** A **function** is a relationship between input and output. In a function, there is exactly one output for each input.

**Key Concept Function**

**Words** A function is a relation in which each element of the domain is paired with exactly one element of the range.

**Examples**



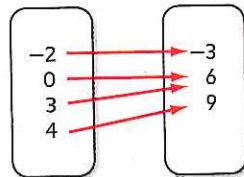
TEKS A.12(A)



**Example 1 Identify Functions**

Determine whether each relation is a function. Explain.

**a. Domain Range**



For each member of the domain, there is only one member of the range. So this mapping represents a function. It does not matter if more than one element of the domain is paired with one element of the range.

**b.**

Domain	1	3	5	1
Range	4	2	4	-4

The element 1 in the domain is paired with both 4 and -4 in the range. So, when  $x$  equals 1 there is more than one possible value for  $y$ . This relation is not a function.

**Guided Practice**

- $\{(2, 1), (3, -2), (3, 1), (2, -2)\}$

A graph that consists of points that are not connected is a **discrete function**.  
 A function graphed with a line or smooth curve is a **continuous function**.

TEKS A.2(A), A.12(A)



**Real-World Example 2** Draw Graphs

**ICE SCULPTING** At an ice sculpting competition, each sculpture's height was measured to make sure that it was within the regulated height range of 0 to 6 feet. The measurements were as follows: Team 1, 4 feet; Team 2, 4.5 feet; Team 3, 3.2 feet; Team 4, 5.1 feet; Team 5, 4.8 feet.

- a. Make a table of values showing the relation between the ice sculpting team and the height of their sculpture.

Team Number	1	2	3	4	5
Height (ft)	4	4.5	3.2	5.1	4.8

- b. State the domain and range of the relation. Then determine whether it is a function.

The domain of the relation is  $\{1, 2, 3, 4, 5\}$  because this set represents values of the independent variable. It is unaffected by the heights.

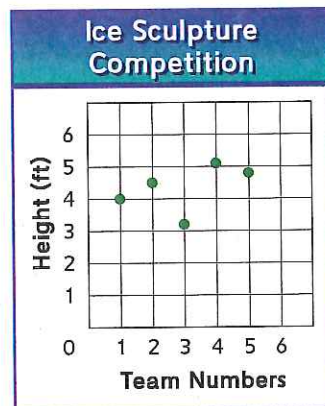
The range of the relation is  $\{4, 4.5, 3.2, 5.1, 4.8\}$  because this set represents values of the dependent variable. This value depends on the team number.

For each member in the domain, there is only one member in the range, so the relation is a function.

- c. Write the data as a set of ordered pairs. Then graph the data.

Use the table. The team number is the independent variable and the height of the sculpture is the dependent variable. Therefore, the ordered pairs are  $(1, 4)$ ,  $(2, 4.5)$ ,  $(3, 3.2)$ ,  $(4, 5.1)$ , and  $(5, 4.8)$ .

Because the team numbers and their corresponding heights cannot be between the points given, the points should not be connected.



- d. State whether the relation is *discrete* or *continuous*. Explain your reasoning.

Because the points are not connected, the relation is discrete.

**Guided Practice**

2. A bird feeder will hold up to 3 quarts of seed. The feeder weighs 2.3 pounds when empty and 13.4 pounds when full.
- Make a table that shows the bird feeder with 0, 1, 2, and 3 quarts of seed in it weighing 2.3, 6, 9.7, 13.4 pounds respectively.
  - State the domain and range of the relation. Then determine whether it is a function.
  - Write the data as a set of ordered pairs. Then graph the data.
  - State whether the relation is *discrete* or *continuous*. Explain your reasoning.



**Real-WorldLink**

The Icehotel, located in the Arctic Circle in Sweden, is a hotel made out of ice. The ice insulates the igloo-like hotel so the temperature is at least  $-8^{\circ}\text{C}$ .

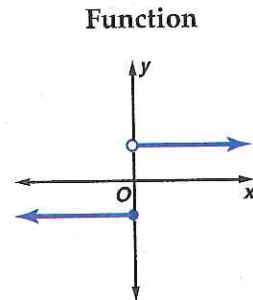
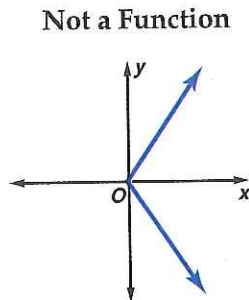
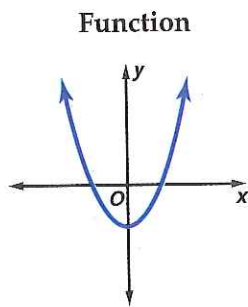
Source: Icehotel

### Study Tip

#### Vertical Line Test

One way to perform the vertical line test is to use a pencil. Place your pencil vertically on the graph and move from left to right. If the pencil passes over the graph in only one place, then the graph represents a function.

You can use the **vertical line test** to see if a graph represents a function. If a vertical line intersects the graph more than once, then the graph is not a function. Otherwise, the relation is a function.



Recall that an equation is a representation of a relation. Equations can also represent functions. Every solution of the equation is represented by a point on a graph. The graph of an equation is the set of all its solutions, which often forms a curve or a line.



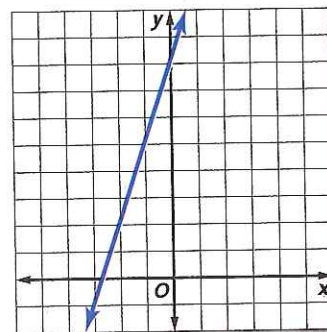
### Example 3 Equations as Functions

Determine whether  $-3x + y = 8$  is a function.

First make a table of values. Then graph the equation.

x	-1	0	1	2
y	5	8	11	14

Connect the points with a smooth graph to represent all of the solutions of the equation. The graph is a line. To use the vertical line test, place a pencil at the left of the graph to represent a vertical line. Slowly move the pencil across the graph.



For any value of  $x$ , the vertical line passes through no more than one point on the graph. So, the graph and the equation represent a function.

**Guided Practice** Determine whether each relation is a function.

3A.  $4x = 8$

3B.  $4x = y + 8$

A function can be represented in different ways.

### Concept Summary Representations of a Function

Table	Mapping	Equation	Graph								
<table border="1"><tr><td>x</td><td>y</td></tr><tr><td>-2</td><td>1</td></tr><tr><td>0</td><td>-1</td></tr><tr><td>2</td><td>1</td></tr></table>	x	y	-2	1	0	-1	2	1	<p>Domain: <math>\{-2, 0, 2\}</math></p> <p>Range: <math>\{1, -1\}</math></p>	$f(x) = \frac{1}{2}x^2 - 1$	
x	y										
-2	1										
0	-1										
2	1										



### StudyTip

#### Function Notation

Functions are indicated by the symbol  $f(x)$ . This is read  $f$  of  $x$ . Other letters, such as  $g$  or  $h$ , can be used to represent functions.

**2 Find Function Values** Equations that are functions can be written in a form called **function notation**. For example, consider  $y = 3x - 8$ .

$$\begin{array}{l} \text{Equation} \\ y = 3x - 8 \end{array}$$

$$\begin{array}{l} \text{Function Notation} \\ f(x) = 3x - 8 \end{array}$$

In a function,  $x$  represents the elements of the domain, and  $f(x)$  represents the elements of the range. The graph of  $f(x)$  is the graph of the equation  $y = f(x)$ . Suppose you want to find the value in the range that corresponds to the element 5 in the domain. This is written  $f(5)$  and is read  $f$  of 5. The value  $f(5)$  is found by substituting 5 for  $x$  in the equation.

TEKS A.12(B)



#### Example 4 Function Values

For  $f(x) = -4x + 7$ , find each value.

a.  $f(2)$

$$\begin{aligned} f(2) &= -4(2) + 7 \\ &= -8 + 7 \\ &= -1 \end{aligned}$$

$$x = 2$$

Multiply.

Add.

b.  $f(-3) + 1$

$$\begin{aligned} f(-3) + 1 &= [-4(-3) + 7] + 1 \\ &= 19 + 1 \\ &= 20 \end{aligned}$$

$$x = -3$$

Simplify.

Add.

#### Guided Practice

For  $f(x) = 2x - 3$ , find each value.

4A.  $f(1)$

4B.  $6 - f(5)$

4C.  $f(-2)$

4D.  $f(-1) + f(2)$

A function with a graph that is not a straight line is a **nonlinear function**.

TEKS A.12(B)



### Go Online!



Use the **Self-Check Quiz** in ConnectED to check your own progress as you complete each lesson.

#### Self-Check Quizzes

Lesson 1-7  
Functions

1.  $h(f) = 2x + 5$ , find  $h(2)$ .

O.A. 10

O.B. 27

O.C. 11

O.D. 5

Use

#### Example 5 Nonlinear Function Values

If  $h(t) = -16t^2 + 68t + 2$ , find each value.

a.  $h(4)$

$$\begin{aligned} h(4) &= -16(4)^2 + 68(4) + 2 \\ &= -256 + 272 + 2 \\ &= 18 \end{aligned}$$

Replace  $t$  with 4.

Multiply.

Add.

b.  $2[h(g)]$

$$\begin{aligned} 2[h(g)] &= 2[-16(g)^2 + 68(g) + 2] \\ &= 2(-16g^2 + 68g + 2) \\ &= -32g^2 + 136g + 4 \end{aligned}$$

Replace  $t$  with  $g$ .

Simplify.

Distributive Property

#### Guided Practice

If  $f(t) = 2t^3$ , find each value.

5A.  $f(4)$

5B.  $3[f(t)] + 2$

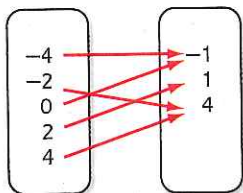
5C.  $f(-5)$

5D.  $f(-3) - f(1)$

**Examples 1, 3** Determine whether each relation is a function. Explain.

TEKS A.12(A)

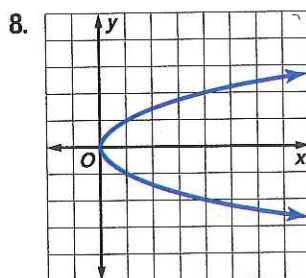
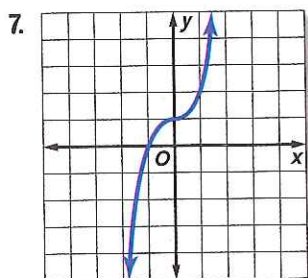
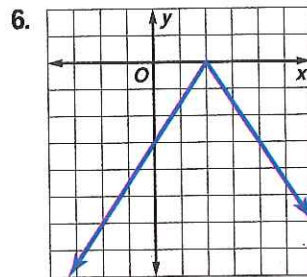
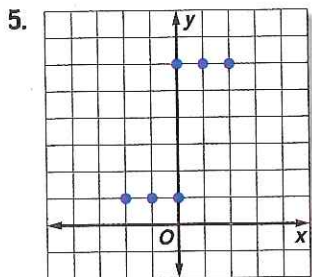
1. Domain Range



Domain	Range
2	6
5	7
6	9
6	10

3.  $\{(2, 2), (-1, 5), (5, 2), (2, -4)\}$

4.  $y = \frac{1}{2}x - 6$



**Example 2**

TEKS A.2(A), A.12(A)

9. **SCHOOL ENROLLMENT** The table shows the total enrollment in Texas public schools.

School Year	2008-09	2009-10	2010-11	2011-12
Enrollment (in thousands)	4750	4848	4934	4999

Source: TEA

- Write a set of ordered pairs representing the data in the table if  $x$  is the number of school years since 2008-2009.
  - Draw a graph showing the relationship between the year and enrollment.
  - Describe the domain and range of the data. Is the relation a function?
10. **MP ORGANIZE IDEAS** The cost of printing digital pictures is given by  $y = 0.13x$ , where  $x$  is the number of pictures that you print and  $y$  is the cost in dollars.
- Determine the domain and range of this relation. Why is the relation a function?
  - Write the equation in function notation. Interpret the function in terms of the context.
  - Find  $f(5)$  and  $f(12)$ . What do these values represent?

**Examples 4-5** If  $f(x) = 6x + 7$  and  $g(x) = x^2 - 4$ , find each value.

TEKS A.12(B)

11.  $f(-3)$

12.  $f(m)$

13.  $f(r - 2)$

14.  $g(5)$

15.  $g(a) + 9$

16.  $g(-4t)$

17.  $f(q + 1)$

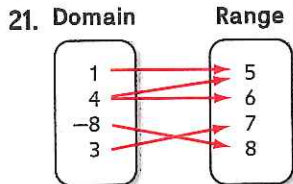
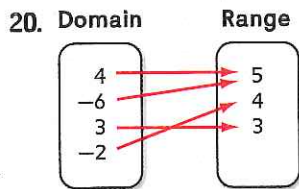
18.  $f(2) + g(2)$

19.  $g(-b)$

**Example 1**

Determine whether each relation is a function. Explain.

**TEKS** A.12(A)

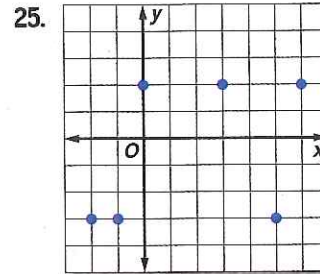
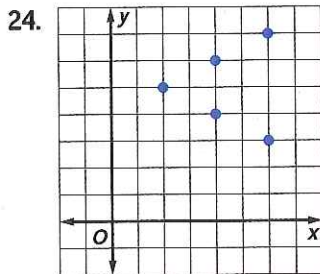


22.

Domain	Range
4	6
-5	3
6	-3
-5	5

23.

Domain	Range
-4	2
3	-5
4	2
9	-7
-3	-5



**Example 2**

26. **MP ORGANIZE IDEAS** The table shows the median home prices in the United States, from 2011 to 2013.

**TEKS** A.2(A), A.12(A)

Year	Median Home Price (\$)
2011	175,600
2012	180,300
2013	208,000

- Write a set of ordered pairs representing the data in the table.
- Draw a graph showing the relationship between the year and price.
- What is the domain and range for this data? Is the relation a function?

**Example 3**

Determine whether each relation is a function.

**TEKS** A.12(A)

- $\{(5, -7), (6, -7), (-8, -1), (0, -1)\}$
- $\{(4, 5), (3, -2), (-2, 5), (4, 7)\}$
- $y = -8$
- $x = 15$
- $y = 3x - 2$
- $y = 3x + 2y$

**Examples 4-5** If  $f(x) = -2x - 3$  and  $g(x) = x^2 + 5x$ , find each value.

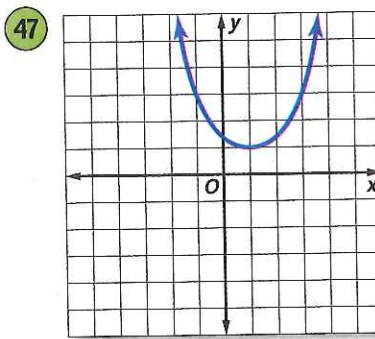
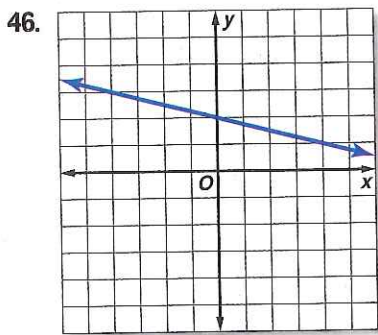
**TEKS** A.12(B)

- $f(-1)$
- $f(6)$
- $g(2)$
- $g(-3)$
- $g(-2) + 2$
- $f(0) - 7$
- $f(4y)$
- $g(-6m)$
- $f(c - 5)$
- $f(r + 2)$
- $5[f(d)]$
- $3[g(n)]$

**45 EDUCATION** Mr. Blankenship determined that his class averages  $f(z)$  can be represented as a function of the class size  $z$  by  $f(z) = -\frac{5}{8}z + 87$ .

- Graph this function. Interpret the function in terms of the context.
- What is the class size that corresponds to a class average of 72?
- What is the domain and range of this function?

Determine whether each relation is a function.

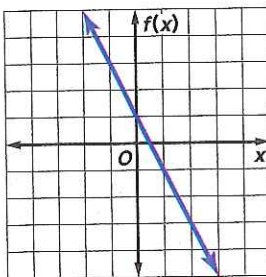


48. **BABYSITTING** Christina earns \$7.50 an hour babysitting.
- Write an algebraic expression to represent the money Christina will earn if she works  $h$  hours.
  - Choose five values for the number of hours Christina can babysit. Create a table with  $h$  and the amount of money she will make during that time.
  - Use the values in your table to create a graph.
  - Does it make sense to connect the points in your graph with a line? Why or why not?

TEKS A.12(A), A.12(B)

**H.O.T. Problems** Use Higher-Order Thinking Skills

49. **MP ORGANIZE IDEAS** Write a set of three ordered pairs that represent a function. Choose another display that represents this function.
50. **MP ANALYZE RELATIONSHIPS** The set of ordered pairs  $\{(0, 1), (3, 2), (3, -5), (5, 4)\}$  represents a relation between  $x$  and  $y$ . Graph the set of ordered pairs. Determine whether the relation is a function. Explain.
51. **MP PROBLEM SOLVING** Consider  $f(x) = -4.3x - 2$ . Write  $f(g + 3.5)$  and simplify by combining like terms.
52. **WRITE A QUESTION** A classmate graphed a set of ordered pairs and used the vertical line test to determine whether it was a function. Write a question to help her decide if the same strategy can be applied to a mapping.
53. **MP APPLY MATH** If  $f(3b - 1) = 9b - 1$ , find one possible expression for  $f(x)$ .
54. **ERROR ANALYSIS** Corazon thinks  $f(x)$  and  $g(x)$  are representations of the same function. Maggie disagrees. Who is correct? Explain your reasoning.



$x$	$g(x)$
-1	1
0	-1
1	-3
2	-5
3	-7

55. **WRITING IN MATH** How can you determine whether a relation represents a function?



## Example

TEKS A.12(B) MP A.1(D)

**GRIDDABLE** If  $f(x) = 1 + 2x - 3x^2$ , what is the value  $f(-7)$ ?

Replace the variable with  $-7$  and evaluate the function.

$$f(x) = 1 + 2x - 3x^2 \quad \text{Original function}$$

$$f(-7) = 1 + 2(-7) - 3(-7)^2 \quad \text{Replace } x \text{ with } -7.$$

$$= 1 - 14 - 3(49) \quad \text{Simplify.}$$

$$= 1 - 14 - 147 \quad \text{Multiply.}$$

$$= -160 \quad \text{Subtract.}$$

The value of the function is  $-160$ . Write  $-160$  at the top of the grid and fill in the appropriate bubbles.

-					1	6	0
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
	1	1	1	1	<input checked="" type="radio"/>	1	1
	2	2	2	2	2	2	2
	3	3	3	3	3	3	3
	4	4	4	4	4	4	4
	5	5	5	5	5	5	5
	6	6	6	6	6	<input checked="" type="radio"/>	6
	7	7	7	7	7	7	7
	8	8	8	8	8	8	8
	9	9	9	9	9	9	9

56. A student collected data on the cost of T-shirts at six stores. The table shown represents the price paid  $p$  for the number of shirts purchased  $n$ .

$n$	5	3	5	3	4	6
$p$	35	24	36	21	28	40

For the six stores, is the price paid a function of the number of shirts purchased? **TEKS** A.12(A)

**MP** A.1(A), A.1(G)

- A Yes, because different people shopped at different stores.
- B Yes, because the total cost was more than one dollar per shirt.
- C No, because there are different total costs for the same number of shirts.
- D No, because the total price is not related to the number of items purchased.

57. **GRIDDABLE** If  $g(x) = \frac{x^2 - 3}{10}$ , what is the value of  $g(1)$ ? **TEKS** A.12(B) **MP** A.1(D)

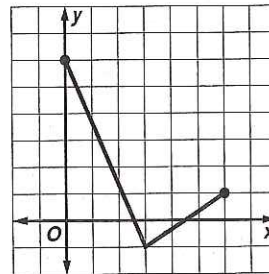
58. For the function  $y = 15x - 4$ , assume the domain is only values of  $x$  from 0 to 5. What is the range of the function? **TEKS** A.2(A) **MP** A.1(F)

- F All values from 15 to 20
- G All values from  $\frac{4}{15}$  to  $\frac{3}{5}$
- H All values from  $-4$  to  $71$
- J The two values  $-4$  and  $71$

59. Which statement best describes how to determine when a graph represents a function? **TEKS** A.12(A) **MP** A.1(E)

- A At least one vertical line intersects the function.
- B Every horizontal line intersects the function.
- C Every vertical line intersects the function exactly one time.
- D Every vertical line intersects the function no more than one time.

60. Which of the following best describes the relation shown in the graph? **TEKS** A.2(A), A.12(A) **MP** A.1(G)



- F Domain:  $0 \leq x \leq 6$ ; Range:  $-1 \leq y \leq 6$ ; the relation is a function
- G Domain:  $0 \leq x \leq 6$ ; Range:  $-1 \leq y \leq 6$ ; the relation is not a function
- H Domain:  $-1 \leq x \leq 6$ ; Range:  $0 \leq y \leq 6$ ; the relation is a function
- J Domain:  $-1 \leq x \leq 6$ ; Range:  $0 \leq y \leq 6$ ; the relation is not a function

# EXTEND 1-7

## Graphing Technology Lab Representing Functions



You can use TI-Nspire Technology to explore the different ways to represent a function.



### Targeted TEKS

**A.3(C)** Graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

### Mathematical Processes



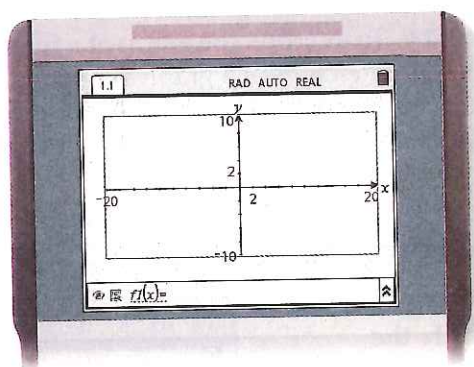
**A.1(C)** Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.



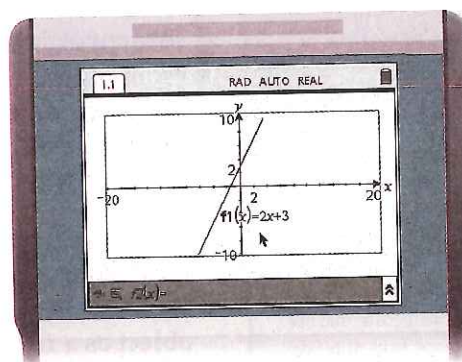
### Activity

Work cooperatively. Graph  $f(x) = 2x + 3$  on the TI-Nspire graphing calculator.

**Step 1** Add a new Graphs page.

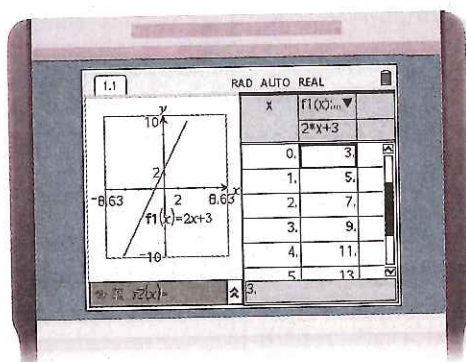


**Step 2** Enter  $2x + 3$  in the entry line.

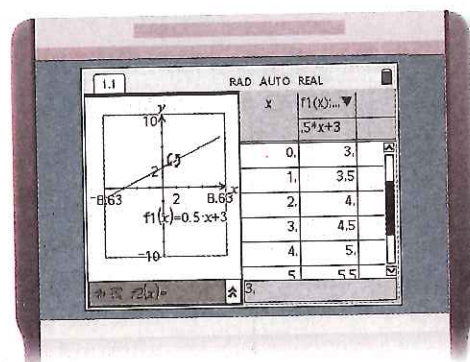


Represent the function as a table.

**Step 3** Select the Show Table option from the View menu to add a table of values on the same display.



**Step 4** Press ctrl and tab to toggle from the table to the graph. On the graph side, select the line and move it. Notice how the values in the table change.



### Cooperatively Analyze the Results

**MP TOOLS AND TECHNIQUES** Graph each function. Make a table of five ordered pairs that also represents the function.

1.  $g(x) = -x - 3$

2.  $h(x) = \frac{1}{3}x + 3$

3.  $f(x) = -\frac{1}{2}x - 5$

4.  $f(x) = 3x - \frac{1}{2}$

5.  $g(x) = -2x + 5$

6.  $h(x) = \frac{1}{5}x + 4$

# 1-8 Interpreting Graphs of Functions

**Then**

- You identified functions and found function values.

**Now**

- 1 Interpret intercepts, and symmetry of graphs of functions.
- 2 Interpret positive, negative, increasing, and decreasing behavior, extrema, and end behavior of graphs of functions.

**Why?**

- Sales of video games, including hardware, software, and accessories, have increased at times and decreased at other times over the years. Annual retail video game sales in the U.S. from 2000 to 2009 can be modeled by the graph of a nonlinear function.



**Targeted TEKS**

**A.3(C)** Graph linear functions on the coordinate plane and identify key features, including  $x$ -intercept,  $y$ -intercept, zeros, and slope, in mathematical and real-world problems.

**A.7(A)** Graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including  $x$ -intercept,  $y$ -intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.



**Mathematical Processes**

**A.1(F)** Analyze mathematical relationships to connect and communicate mathematical ideas.

**A.1(G)** Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.



**New Vocabulary**

- intercept
- $x$ - and  $y$ -intercepts
- line
- symmetry
- positive
- negative
- increasing
- decreasing
- extrema
- relative maximum
- relative minimum
- end behavior

**1 Interpret Intercepts and Symmetry** To interpret the graph of a function, estimate and interpret key features. The **intercepts** of a graph are points where the graph intersects an axis. The  $y$ -coordinate of the point at which the graph intersects the  $y$ -axis is called a  **$y$ -intercept**. Similarly, the  $x$ -coordinate of the point at which a graph intersects the  $x$ -axis is called an  **$x$ -intercept**.

TEKS A.7(A)



**Real-World Example 1** Interpret Intercepts

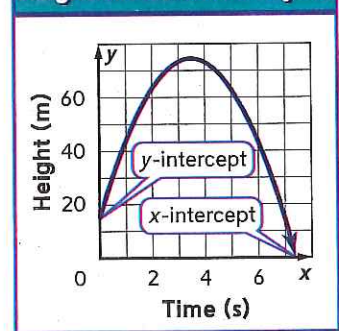
**PHYSICS** The graph shows the height  $y$  of an object as a function of time  $x$ . Identify the function as *linear* or *nonlinear*. Then estimate and interpret the intercepts.

**Linear or Nonlinear:** Since the graph is a curve and not a line, the graph is nonlinear.

**$y$ -Intercept:** The graph intersects the  $y$ -axis at about  $(0, 15)$ , so the  $y$ -intercept of the graph is about 15. This means that the object started at an initial height of about 15 meters above the ground.

**$x$ -Intercept(s):** The graph intersects the  $x$ -axis at about  $(7.4, 0)$ , so the  $x$ -intercept is about 7.4. This means that the object struck the ground after about 7.4 seconds.

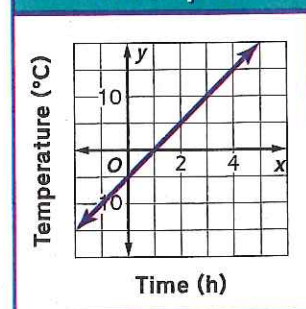
Height of Launched Object



**Guided Practice**

1. The graph shows the temperature  $y$  of a medical sample thawed at a controlled rate. Identify the function as *linear* or *nonlinear*. Then estimate and interpret the intercepts.

Controlled Thaw of Sample



A graph possesses **line symmetry** in the  $y$ -axis or some other vertical line if each half of the graph on either side of the line matches exactly. The graphs of most real-world functions do not exhibit symmetry over the entire domain. However, many have symmetry over smaller portions of the domain that are worth analyzing.

TEKS A.7(A)



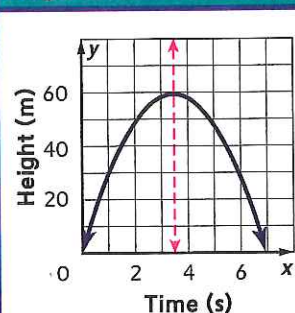
### Real-World Example 2 Interpret Symmetry

**PHYSICS** An object is launched. The graph shows the height  $y$  of the object as a function of time  $x$ . Describe and interpret any symmetry.

The right half of the graph is the mirror image of the left half in approximately the line  $x = 3.5$  between approximately  $x = 0$  and  $x = 7$ .

In the context of the situation, the symmetry of the graph tells you that the time it took the object to go up is equal to the time it took to come down.

Height of Launched Object



#### Guided Practice

- Describe and interpret any symmetry exhibited by the graph in Guided Practice 1.

### Go Online!

Taking notes in your **eStudent Edition** allows you to keep your notes with each lesson to reference anytime.

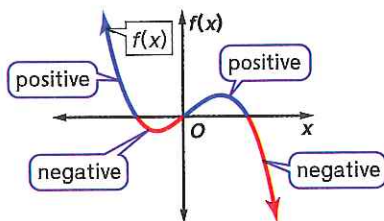


## 2 Interpret Extrema and End Behavior

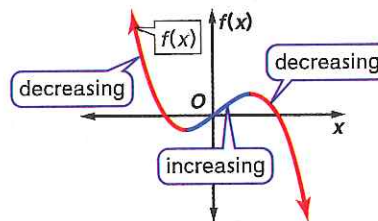
Interpreting a graph also involves estimating and interpreting where the function is increasing, decreasing, positive, or negative, and where the function has any extreme values, either high or low.

### Key Concepts Positive, Negative, Increasing, Decreasing, Extrema, and End Behavior

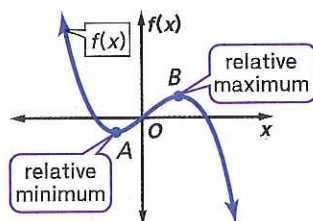
A function is **positive** where its graph lies above the  $x$ -axis, and **negative** where its graph lies below the  $x$ -axis.



A function is **increasing** where the graph goes up and **decreasing** where the graph goes down when viewed from left to right.

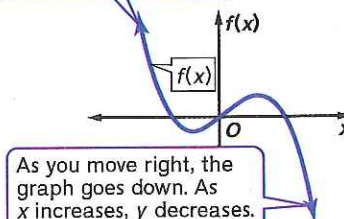


The points shown are the locations of relatively high or low function values called **extrema**. Point A is a **relative minimum**, since no other nearby points have a lesser  $y$ -coordinate. Point B is a **relative maximum**, since no other nearby points have a greater  $y$ -coordinate.



**End behavior** describes the values of a function at the positive and negative extremes in its domain.

As you move left, the graph goes up. As  $x$  decreases,  $y$  increases.



As you move right, the graph goes down. As  $x$  increases,  $y$  decreases.

### StudyTip

**End Behavior** The end behavior of some graphs can be described as approaching a specific  $y$ -value. In this case, a portion of the graph looks like a horizontal line.





**Real-WorldLink**

The first successful commercially sold portable video game system was released in 1989 and sold for \$120.

Source: PCWorld

**StudyTip**

**Constant** A function is constant where the graph does not go up or down as the graph is viewed from left to right.

**VIDEO GAMES** U.S. retail sales of video games from 2000 to 2009 can be modeled by the function graphed at the right. Estimate and interpret where the function is positive, negative, increasing, and decreasing, the  $x$ -coordinates of any relative extrema, and the end behavior of the graph.



**Positive:** between about  $x = -0.6$  and  $x = 10.4$

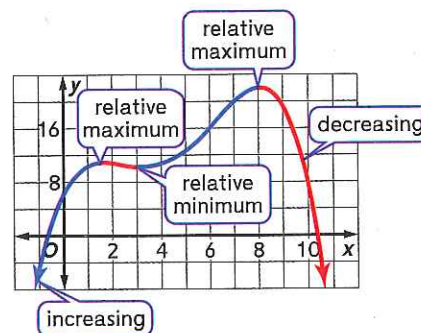
**Negative:** for about  $x < -0.6$  and  $x > 10.4$

This means that there were positive sales between about 2000 and 2010, but the model predicts negative sales after about 2010, indicating the unlikely collapse of the industry.

**Increasing:** for about  $x < 1.5$  and between about  $x = 3$  and  $x = 8$

**Decreasing:** between about  $x = 2$  and  $x = 3$  and for about  $x > 8$

This means that sales increased from about 2000 to 2002, decreased from 2002 to 2003, increased from 2003 to 2008, and have been decreasing since 2008.



**Relative Maximums:** at about  $x = 1.5$  and  $x = 8$

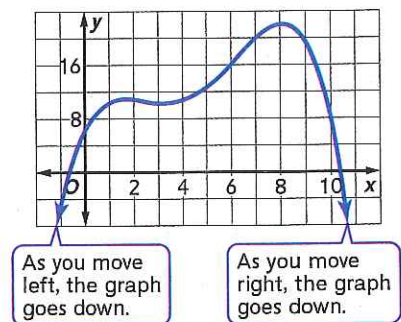
**Relative Minimum:** at about  $x = 3$

The extrema of the graph indicate that the industry experienced two relative peaks in sales during this period: one around 2002 of approximately \$10.5 billion and another around 2008 of approximately \$22 billion. A relative low of \$10 billion in sales came in about 2003.

**End Behavior:**

As  $x$  increases or decreases, the value of  $y$  decreases.

The end behavior of the graph indicates negative sales several years prior to 2000 and several years after 2009, which is unlikely. This graph appears to only model sales well between 2000 and 2009 and can only be used to predict sales in 2010.

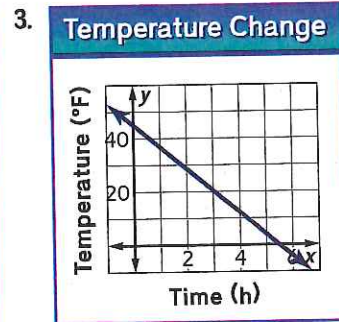
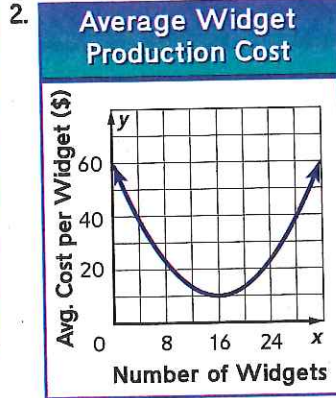
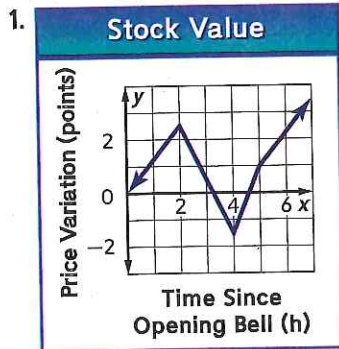


**GuidedPractice**

3. Estimate and interpret where the function graphed in Guided Practice 1 is positive, negative, increasing, or decreasing, the  $x$ -coordinate of any relative extrema, and the end behavior of the graph.

**Examples 1–3** **MP ORGANIZE IDEAS** Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the *x*-coordinate of any relative extrema, and the end behavior of the graph.

**TEKS** A.3(C),  
A.7(A)

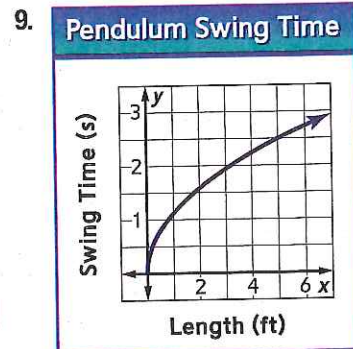
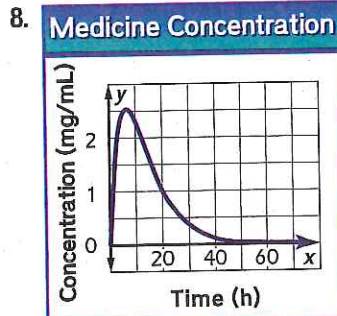
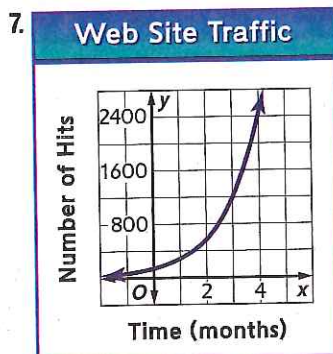
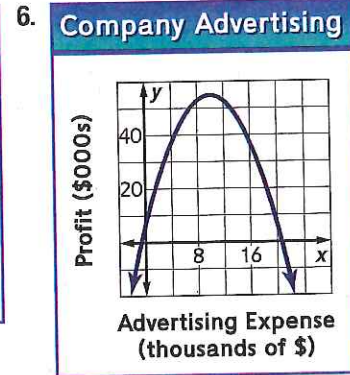
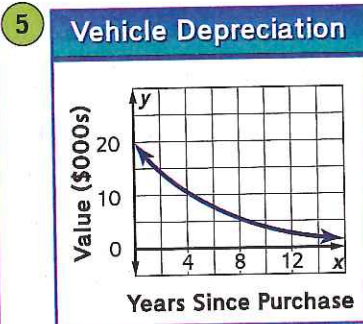
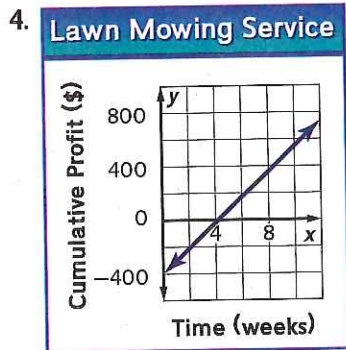


Practice and Problem Solving

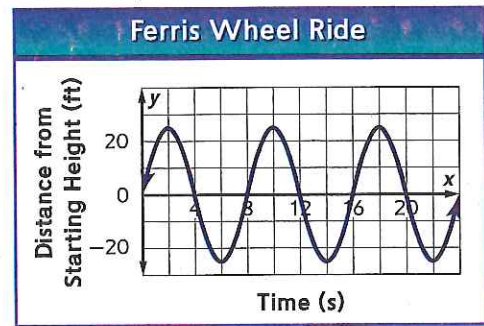
Extra Practice is on page R1.

**Examples 1–3** **MP ORGANIZE IDEAS** Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the *x*-coordinate of any relative extrema, and the end behavior of the graph.

**TEKS** A.3(C),  
A.7(A)



10. **FERRIS WHEEL** At the beginning of a Ferris wheel ride, a passenger cart is located at the same height as the center of the wheel. The position  $y$  in feet of this cart relative to the center  $t$  seconds after the ride starts is given by the function graphed at the right. Identify and interpret the key features of the graph. (*Hint: Look for a pattern in the graph to help you describe its end behavior.*)



Sketch a graph of a function that could represent each situation. Identify and interpret the intercepts of the graph, where the graph is increasing and decreasing, and any relative extrema.



11. the height of a corn plant from the time the seed is planted until it reaches maturity 120 days later
12. the height of a football from the time it is punted until it reaches the ground 2.8 seconds later
13. the balance due on a car loan from the date the car was purchased until it was sold 4 years later

Sketch graphs of functions with the following characteristics.

14. The graph is linear with an  $x$ -intercept at  $-2$ . The graph is positive for  $x < -2$ , and negative for  $x > -2$ .
15. A nonlinear graph has  $x$ -intercepts at  $-2$  and  $2$  and a  $y$ -intercept at  $-4$ . The graph has a relative minimum of  $-4$  at  $x = 0$ . The graph is decreasing for  $x < 0$  and increasing for  $x > 0$ .
16. A nonlinear graph has a  $y$ -intercept at  $2$ , but no  $x$ -intercepts. The graph is positive and increasing for all values of  $x$ .
17. A nonlinear graph has  $x$ -intercepts at  $-8$  and  $-2$  and a  $y$ -intercept at  $3$ . The graph has relative minimums at  $x = -6$  and  $x = 6$  and a relative maximum at  $x = 2$ . The graph is positive for  $x < -8$  and  $x > -2$  and negative between  $x = -8$  and  $x = -2$ . As  $x$  decreases,  $y$  increases and as  $x$  increases,  $y$  increases.

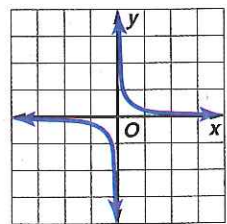
TEKS A.3(C)

### H.O.T. Problems

Use Higher-Order Thinking Skills

18. **MP JUSTIFY ARGUMENTS** Katara thinks that all linear functions have exactly one  $x$ -intercept. Desmond thinks that a linear function can have at most one  $x$ -intercept. Is either of them correct? Explain your reasoning.
19. **MP ANALYZE RELATIONSHIPS** Describe the end behavior of the graph shown.
20. **MP JUSTIFY ARGUMENTS** Determine whether the following statement is *true* or *false*. Explain.

*Functions have at most one  $y$ -intercept.*



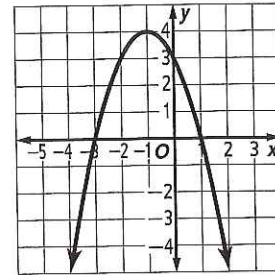
21. **MP ORGANIZE IDEAS** Sketch the graph of a function with one relative maximum and one relative minimum that could represent a real-world function. Label each axis and include appropriate units. Then identify and interpret the relative extrema of your graph.
22. **WRITING IN MATH** Describe how you would identify the key features of a graph described in this lesson using a table of values for a function.

## Example

TEKS A.7(A) MP A.1(E)

Which of the following best describes the graph?

- A The  $x$ -intercepts are  $-3$  and  $1$ , the  $y$ -intercept is  $3$ , and the axis of symmetry is  $y = 4$ .
- B The  $x$ -intercepts are  $-3$  and  $1$ , the  $y$ -intercept is  $3$ , and the axis of symmetry is  $x = -1$ .
- C The  $x$ -intercepts are  $-1$  and  $3$ , the  $y$ -intercept is  $3$ , and the axis of symmetry is  $y = 4$ .
- D The  $x$ -intercepts are  $-3$  and  $1$ , the  $y$ -intercept is  $4$ , and the axis of symmetry is  $x = -1$ .



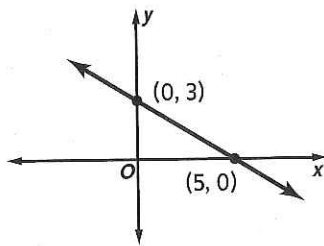
The  $x$ -intercepts are located where the graph crosses the  $x$ -axis. This occurs at  $x = -3$  and  $x = 1$ .

The  $y$ -intercept is located where the graph crosses the  $y$ -axis. This occurs at  $y = 3$ .

The axis of symmetry is the vertical line through the vertex  $(-1, 4)$ , so its equation is  $x = -1$ .

The correct answer is choice B.

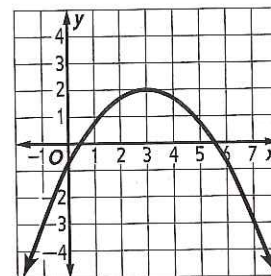
23. Which of the following best describes the graph? TEKS A.3(C) MP A.1(E)



- A The  $x$ -intercept is  $3$ ; the  $y$ -intercept is  $5$ ; the graph is positive for  $x < 5$ ; the graph is negative for  $x > 5$ .
- B The  $x$ -intercept is  $5$ ; the  $y$ -intercept is  $3$ ; the graph is positive for  $x < 5$ ; the graph is negative for  $x > 5$ .
- C The  $x$ -intercept is  $5$ ; the  $y$ -intercept is  $3$ ; the graph is positive for  $x > 5$ ; the graph is negative for  $x < 5$ .
- D The  $x$ -intercept is  $5$ ; the  $y$ -intercept is  $3$ ; the graph is positive for  $x > 0$ ; the graph is negative for  $x < 0$ .

24. **GRIDDABLE** Thomas graphs the function  $y = 3(x - 6)^2$ . What is the  $x$ -intercept of the function? TEKS A.7(A) MP A.1(D)

25. Which statement best describes the graph shown? TEKS A.3(C) MP A.1(E)



- F The graph is linear.
  - G The graph is nonlinear.
  - H There are two  $y$ -intercepts.
  - J The graph is increasing.
26. A quadratic function has a negative coefficient on its  $x^2$  term, and the vertex is in the third quadrant. Which of the following best describes the graph of the function? TEKS A.7(A) MP A.1(E)
- A The graph has no  $x$ -intercepts.
  - B The graph has 1  $x$ -intercept.
  - C The graph has 2  $x$ -intercepts.
  - D There is not enough information to determine the number of  $x$ -intercepts.



Go Online! for Vocabulary Review Games and key vocabulary in 13 languages

## Study Guide

### Key Concepts

#### Order of Operations (Lesson 1-2)

- Evaluate expressions inside grouping symbols.
- Evaluate all powers.
- Multiply and/or divide in order from left to right.
- Add or subtract in order from left to right.

#### Properties of Equality (Lessons 1-3 and 1-4)

- For any numbers  $a$ ,  $b$ , and  $c$ :

Reflexive:  $a = a$

Symmetric: If  $a = b$ , then  $b = a$ .

Transitive: If  $a = b$  and  $b = c$ , then  $a = c$ .

Substitution: If  $a = b$ , then  $a$  may be replaced by  $b$  in any expression.

Distributive:  $a(b + c) = ab + ac$   
 $a(b - c) = ab - ac$

Commutative:  $a + b = b + a$  and  $ab = ba$

Associative:  $(a + b) + c = a + (b + c)$  and  
 $(ab)c = a(bc)$

#### Solving Equations (Lesson 1-5)

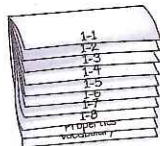
- Apply order of operations and the properties of real numbers to solve equations.

#### Relations, Functions, and Interpreting Graphs of Functions (Lessons 1-6 through 1-8)

- Relations and functions can be represented by ordered pairs, a table, a mapping, or a graph.
- Use the vertical line test to determine if a relation is a function.
- End behavior describes the long-term behavior of a function on either end of its graph.
- Points where the graph of a function crosses an axis are called intercepts.
- A function is positive on a portion of its domain where its graph lies above the  $x$ -axis, and negative on a portion where its graph lies below the  $x$ -axis.

### FOLDABLES® Study Organizer

Use your Foldable to review the chapter. Working with a partner can be helpful. Ask for clarification of concepts as needed.



### Key Vocabulary

- |                              |                             |
|------------------------------|-----------------------------|
| algebraic expression (p. 5)  | ordered pair (p. 40)        |
| base (p. 5)                  | order of operations (p. 10) |
| coefficient (p. 28)          | origin (p. 40)              |
| coordinate system (p. 40)    | power (p. 5)                |
| dependent variable (p. 42)   | range (p. 40)               |
| domain (p. 40)               | reciprocal (p. 17)          |
| end behavior (p. 57)         | relation (p. 40)            |
| equation (p. 33)             | relative maximum (p. 57)    |
| exponent (p. 5)              | relative minimum (p. 57)    |
| function (p. 47)             | replacement set (p. 33)     |
| independent variable (p. 42) | simplest form (p. 27)       |
| intercept (p. 56)            | solution (p. 33)            |
| like terms (p. 27)           | term (p. 5)                 |
| line symmetry (p. 57)        | variables (p. 5)            |
| mapping (p. 40)              | vertical line test (p. 49)  |

### Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. A coordinate system is formed by two intersecting number lines.
2. An exponent indicates the number of times the base is to be used as a factor.
3. An expression is in simplest form when it contains like terms and parentheses.
4. In an expression involving multiplication, the quantities being multiplied are called factors.
5. In a function, there is exactly one output for each input.
6. Order of operations tells us to perform multiplication before subtraction.
7. Since the product of any number and 1 is equal to the number, 1 is called the multiplicative inverse.

# Lesson-by-Lesson Review



## 1-1 Variables and Expressions

TEKS Preparation for A.2(C)

Write a verbal expression for each algebraic expression.

8.  $h - 7$       9.  $3x^2$       10.  $5 + 6m^3$

Write an algebraic expression for each verbal expression.

11. a number increased by 9  
 12. two thirds of a number  $d$  to the third power  
 13. 5 less than four times a number

Evaluate each expression.

14.  $2^5$       15.  $6^3$       16.  $4^4$

17. **BOWLING** Fantastic Pins Bowling Alley charges \$4.75 for shoe rental plus \$4.25 for each game. Write an expression representing the cost to rent shoes and bowl  $g$  games.

### Example 1

Write a verbal expression for  $4x + 9$ .  
 nine more than four times a number  $x$

### Example 2

Write an algebraic expression for *the difference of twelve and two times a number cubed*.

**Variable** Let  $x$  represent the number.

**Expression**  $12 - 2x^3$

### Example 3

Evaluate  $3^4$ .

The base is 3 and the exponent is 4.

$$\begin{aligned} 3^4 &= 3 \cdot 3 \cdot 3 \cdot 3 && \text{Use 3 as a factor 4 times.} \\ &= 81 && \text{Multiply.} \end{aligned}$$

## 1-2 Order of Operations

TEKS Preparation for A.5(A)

Evaluate each expression.

18.  $24 - 4 \cdot 5$       19.  $15 + 3^2 - 6$   
 20.  $7 + 2(9 - 3)$       21.  $8 \cdot 4 - 6 \cdot 5$   
 22.  $[(2^5 - 5) \div 9]11$       23.  $\frac{11 + 4^2}{5^2 - 4^2}$

Evaluate each expression if  $a = 4$ ,  $b = 3$ , and  $c = 9$ .

24.  $c + 3a$   
 25.  $5b^2 \div c$   
 26.  $(a^2 + 2bc) \div 7$

27. **ICE CREAM** The cost of a one-scoop sundae is \$4.75, and the cost of a two-scoop sundae is \$5.25. Write and evaluate an expression to find the total cost of 3 one-scoop sundaes and 2 two-scoop sundaes.

### Example 4

Evaluate the expression  $3(9 - 5)^2 \div 8$ .

$$\begin{aligned} 3(9 - 5)^2 \div 8 &= 3(4)^2 \div 8 && \text{Work inside parentheses.} \\ &= 3(16) \div 8 && \text{Evaluate } 4^2. \\ &= 48 \div 8 && \text{Multiply.} \\ &= 6 && \text{Divide.} \end{aligned}$$

### Example 5

Evaluate the expression  $(5m - 2n) \div p^2$  if  $m = 8$ ,  $n = 4$ ,  $p = 2$ .

$$\begin{aligned} (5m - 2n) \div p^2 &= (5 \cdot 8 - 2 \cdot 4) \div 2^2 && \text{Replace } m \text{ with 8, } n \text{ with 4,} \\ &= (40 - 8) \div 2^2 && \text{and } p \text{ with 2.} \\ &= 32 \div 2^2 && \text{Multiply.} \\ &= 32 \div 4 && \text{Subtract.} \\ &= 8 && \text{Evaluate } 2^2. \\ & && \text{Divide.} \end{aligned}$$

## 1-3 Properties of Numbers

TEKS Preparation for A.5(A)

Evaluate each expression using properties of numbers. Name the property used in each step.

28.  $18 \cdot 3(1 \div 3)$       29.  $[5 \div (8 - 6)]\frac{2}{5}$   
 30.  $(16 - 4^2) + 9$       31.  $2 \cdot \frac{1}{2} + 4(4 \cdot 2 - 7)$   
 32.  $18 + 41 + 32 + 9$       33.  $7\frac{2}{5} + 5 + 2\frac{3}{5}$   
 34.  $8 \cdot 0.5 \cdot 5$       35.  $5.3 + 2.8 + 3.7 + 6.2$

36. **SCHOOL SUPPLIES** Monica needs to purchase a binder, a textbook, a calculator, and a workbook for her algebra class. The binder costs \$9.25, the textbook \$72.50, the calculator \$49.99, and the workbook \$15.00. Find the total cost for Monica's algebra supplies.

## Example 6

Evaluate  $6(4 \cdot 2 - 7) + 5 \cdot \frac{1}{5}$ . Name the property used in each step.

$$\begin{aligned} 6(4 \cdot 2 - 7) + 5 \cdot \frac{1}{5} & \\ = 6(8 - 7) + 5 \cdot \frac{1}{5} & \quad \text{Substitution} \\ = 6(1) + 5 \cdot \frac{1}{5} & \quad \text{Substitution} \\ = 6 + 5 \cdot \frac{1}{5} & \quad \text{Multiplicative Identity} \\ = 6 + 1 & \quad \text{Multiplicative Inverse} \\ = 7 & \quad \text{Substitution} \end{aligned}$$

## 1-4 The Distributive Property

TEKS A.10(D)

Use the Distributive Property to rewrite each expression. Then evaluate.

37.  $(2 + 3)6$       38.  $5(18 + 12)$   
 39.  $8(6 - 2)$       40.  $(11 - 4)3$   
 41.  $-2(5 - 3)$       42.  $(8 - 3)4$

Rewrite each expression using the Distributive Property. Then simplify.

43.  $3(x + 2)$       44.  $(m + 8)4$   
 45.  $6(d - 3)$       46.  $-4(5 - 2t)$   
 47.  $(9y - 6)(-3)$       48.  $-6(4z + 3)$

49. **TUTORING** Write and evaluate an expression for the number of tutoring lessons Mrs. Green gives in 4 weeks.

Tutoring Schedule	
Day	Students
Monday	3
Tuesday	5
Wednesday	4

## Example 7

Use the Distributive Property to rewrite the expression  $5(3 + 8)$ . Then evaluate.

$$\begin{aligned} 5(3 + 8) &= 5(3) + 5(8) && \text{Distributive Property} \\ &= 15 + 40 && \text{Multiply.} \\ &= 55 && \text{Simplify.} \end{aligned}$$

## Example 8

Rewrite the expression  $6(x + 4)$  using the Distributive Property. Then simplify.

$$\begin{aligned} 6(x + 4) &= 6 \cdot x + 6 \cdot 4 && \text{Distributive Property} \\ &= 6x + 24 && \text{Simplify.} \end{aligned}$$

## Example 9

Rewrite the expression  $(3x - 2)(-5)$  using the Distributive Property. Then simplify.

$$\begin{aligned} (3x - 2)(-5) & \\ = (3x)(-5) - (2)(-5) & \quad \text{Distributive Property} \\ = -15x + 10 & \quad \text{Simplify.} \end{aligned}$$

## 1-5 Equations

TEKS A.5(A)

Find the solution set of each equation if the replacement sets are  $x: \{1, 3, 5, 7, 9\}$  and  $y: \{6, 8, 10, 12, 14\}$ .

50.  $y - 9 = 3$                       51.  $14 + x = 21$   
 52.  $4y = 32$                         53.  $3x - 11 = 16$   
 54.  $\frac{42}{y} = 7$                         55.  $2(x - 1) = 8$

Solve each equation.

56.  $a = 24 - 7(3)$   
 57.  $z = 63 \div (3^2 - 2)$   
 58. **AGE** Shandra's age is four more than three times Sherita's age. Write an equation for Shandra's age. Solve if Sherita is 3 years old

### Example 10

Solve the equation  $5w - 19 = 11$  if the replacement set is  $w: \{2, 4, 6, 8, 10\}$ .

Replace  $w$  in  $5w - 19 = 11$  with each value in the replacement set.

$w$	$5w - 19 = 11$	True or False?
2	$5(2) - 19 = 11$	false
4	$5(4) - 19 = 11$	false
6	$5(6) - 19 = 11$	true
8	$5(8) - 19 = 11$	false
10	$5(10) - 19 = 11$	false

Since the equation is true when  $w = 6$ , the solution of  $5w - 19 = 11$  is  $w = 6$ .

## 1-6 Relations

TEKS A.2(A)

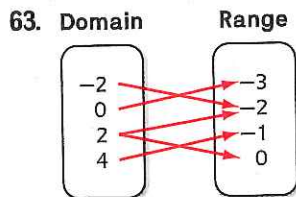
Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

59.  $\{(1, 3), (2, 4), (3, 5), (4, 6)\}$   
 60.  $\{(-1, 1), (0, -2), (3, 1), (4, -1)\}$   
 61.  $\{(-2, 4), (-1, 3), (0, 2), (-1, 2)\}$

Express the relation shown in each table, mapping, or graph as a set of ordered pairs.

62.

$x$	$y$
5	3
3	-1
1	2
-1	0



64. **GARDENING** On average, 7 plants grow for every 10 seeds of a certain type planted. Make a table to show the relation between seeds planted and plants growing for 50, 100, 150, and 200 seeds. Then state the domain and range and graph the relation.

### Example 11

Express the relation  $\{(-3, 4), (1, -2), (0, 1), (3, -1)\}$  as a table, a graph, and a mapping.

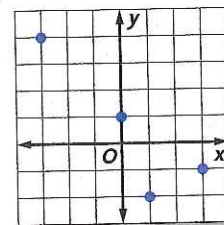
#### Table

Place the  $x$ -coordinates into the first column. Place the corresponding  $y$ -coordinates in the second column.

$x$	$y$
-3	4
1	-2
0	1
3	-1

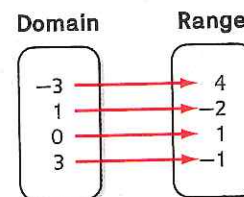
#### Graph

Graph each ordered pair on a coordinate plane.



#### Mapping

List the  $x$ -values in the domain and the  $y$ -values in the range. Draw arrows from the  $x$ -values in set  $X$  to the corresponding  $y$ -values in set  $Y$ .



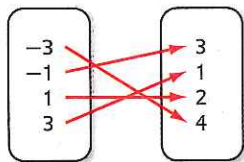


1-7 Functions

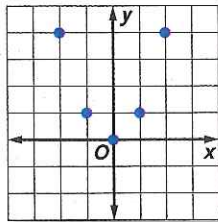
TEKS A.12(A), A.12(B)

Determine whether each relation is a function.

65.



66.



67.  $\{(8, 4), (6, 3), (4, 2), (2, 1), (6, 0)\}$

If  $f(x) = 2x + 4$  and  $g(x) = x^2 - 3$ , find each value.

68.  $f(-3)$

69.  $g(2)$

70.  $f(0)$

71.  $g(-4)$

72.  $f(m + 2)$

73.  $g(3p)$

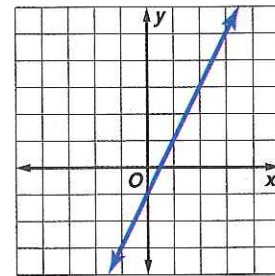
74. **FOOTBALL** After three quarters, Omar had rushed 45 yards. If Omar averages 9 yards per carry in the fourth quarter and there is enough time to run 6 more plays before the end of the game, the equation  $g(x) = 45 + 9x$  can be used to represent the situation, where  $x$  is the number of carries. Graph this function and find its domain and range.

Example 12

Determine whether  $2x - y = 1$  represents a function.

First make a table of values. Then graph the equation.

x	y
-1	-3
0	-1
1	1
2	3
3	5

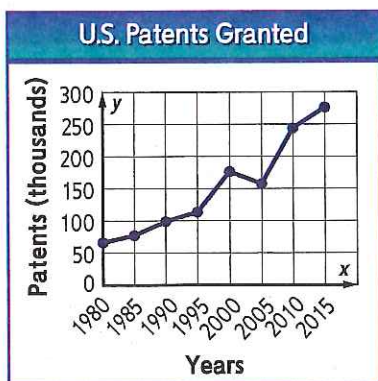


Using the vertical line test, it can be shown that  $2x - y = 1$  does represent a function.

1-8 Interpreting Graphs of Functions

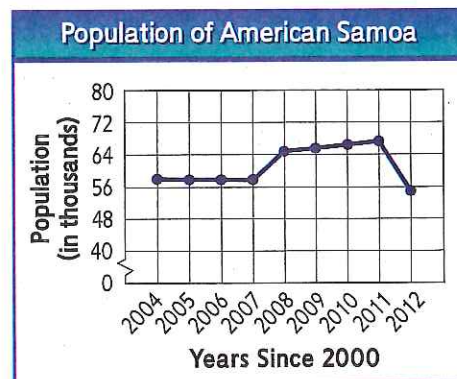
TEKS A.3(C), A.7(A)

75. Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing and decreasing, the  $x$ -coordinate of any relative extrema, and the end behavior of the graph.



Example 13

**POPULATION** The population of American Samoa from 2004 to 2012 can be modeled by the graph shown. Estimate and interpret where the function is increasing and decreasing, the  $x$ -coordinates of any relative extrema, and the end behavior of the graph.



The population remained relatively constant from 2004 to 2007, increased from 2007 to 2011, and decreased from 2011 to 2012. The relative maximum of the graph indicates that the population peaked in 2011. The end behavior indicates a decline in population from 2011 to 2012.



Write an algebraic expression for each verbal expression.

- six more than a number
- twelve less than the product of three and a number
- four divided by the difference between a number and seven

Evaluate each expression.

4.  $32 \div 4 + 2^3 - 3$

5.  $\frac{(2 \cdot 4)^2}{7 + 3^2}$

6. **MULTIPLE CHOICE** Find the value of the expression  $a^2 + 2ab + b^2$  if  $a = 6$  and  $b = 4$ .

- A 68  
B 92  
C 100  
D 121

Evaluate each expression. Name the property used in each step.

7.  $13 + (16 - 4^2)$

8.  $\frac{2}{9}[9 \div (7 - 5)]$

9.  $37 + 29 + 13 + 21$

Rewrite each expression using the Distributive Property. Then simplify.

10.  $4(x + 3)$

11.  $(5p - 2)(-3)$

12. **MOVIE TICKETS** A company operates three movie theaters. The chart shows the typical number of tickets sold each week at the three locations. Write and evaluate an expression for the total typical number of tickets sold by all three locations in four weeks.

Location	Tickets Sold
A	438
B	374
C	512

Find the solution of each equation if the replacement sets are  $x: \{1, 3, 5, 7, 9\}$  and  $y: \{2, 4, 6, 8, 10\}$ .

13.  $3x - 9 = 12$

14.  $y^2 - 5y - 11 = 13$

15. **DOGGIE DAYCARE** A doggie daycare charges a flat registration fee of \$5 plus a \$16 charge per day. Write an equation to find  $C$ , the total cost for  $d$  days. Then solve the equation for  $d = 5$ .

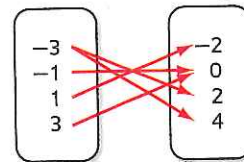
Express the relation shown in each table, mapping, or graph as a set of ordered pairs.

16.

x	y
-2	4
1	2
3	0
4	-2

17. Domain

Range



18. **MULTIPLE CHOICE** Determine the domain and range for the relation  $\{(2, 5), (-1, 3), (0, -1), (3, 3), (-4, -2)\}$ .

F D:  $\{2, -1, 0, 3, -4\}$ , R:  $\{5, 3, -1, 3, -2\}$

G D:  $\{5, 3, -1, 3, -2\}$ , R:  $\{2, -1, 0, 3, 4\}$

H D:  $\{0, 1, 2, 3, 4\}$ , R:  $\{-4, -3, -2, -1, 0\}$

J D:  $\{2, -1, 0, 3, -4\}$ , R:  $\{2, -1, 0, 3, 4\}$

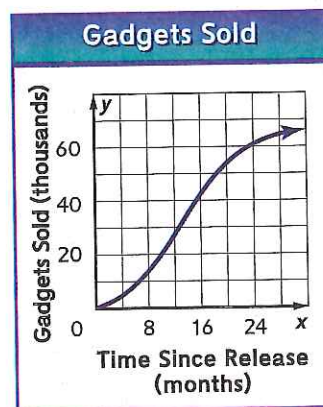
19. Determine whether the relation  $\{(2, 3), (-1, 3), (0, 4), (3, 2), (-2, 3)\}$  is a function.

If  $f(x) = 5 - 2x$  and  $g(x) = x^2 + 7x$ , find each value.

20.  $g(3)$

21.  $f(-6y)$

22. Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the  $x$ -coordinate of any relative extrema, and the end behavior of the graph.



# Preparing for Assessment

## Test-Taking Strategies

### Eliminate Unreasonable Answers

You can eliminate unreasonable answers to help you find the correct one when solving multiple choice test items. Doing so will save you time by narrowing down the list of possible correct answers.

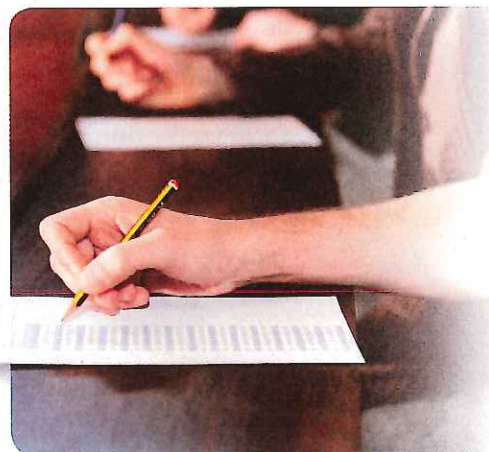
#### Strategies for Eliminating Unreasonable Answers

##### Step 1

Read the problem statement carefully to determine exactly what you are being asked to find.

##### Ask yourself:

- What am I being asked to solve?
- What format (i.e., fraction, number, decimal, percent, type of graph) will the correct answer be?
- What units (if any) will the correct answer have?



##### Step 2

Carefully look over each possible answer choice and evaluate for reasonableness.

- Identify any answer choices that are clearly incorrect and eliminate them.
- Eliminate any answer choices that are not in the proper format.
- Eliminate any answer choices that do not have the correct units.

##### Step 3

Solve the problem and choose the correct answer from those remaining. Check your answer.

#### Texas Assessment Example

TEKS A.2(A) MP A.1(E)

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

What is the domain of the following relation?

$\{(1, 3), (-6, 4), (8, 5)\}$

A  $\{(3, 4, 5)\}$

C  $\{-6, 1, 3, 4, 5, 8\}$

B  $\{-6, 1, 8\}$

D  $\{1, 3, 4, 5, 8\}$

The domain of a relation is the set of  $x$ -coordinates of the ordered pairs in the relation. Look at the  $x$ -coordinates of each ordered pair. Each is unique. So, since there are 3 ordered pairs, the domain will be a set of 3 numbers. Answer choices C and D can be eliminated right away since they contain more elements than needed.

The  $x$ -coordinates of the relation are 1,  $-6$ , and 8. Choice B is the correct answer.

## Exercises

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

1. Refer to the relation in the table below.

$x$	$-6$	$-2$	$0$	$?$	$3$	$5$
$y$	$-1$	$8$	$3$	$-3$	$4$	$0$

Which of the following values would result in the relation not being a function? **TEKS** A.2(A) **MP** A.1(E)

- A  $-1$                                       C  $7$   
 B  $3$                                          D  $8$
2. There are 24 more cars than twice the number of trucks for sale at a local dealership. Which of the following best represents this situation?

**TEKS** A.2(C) **MP** A.1(A)

- F  $C = 2T + 24$   
 G  $T = 2C + 24$   
 H  $C = T + 2$   
 J  $T = C + 24$
3. What is the range of the relation below?

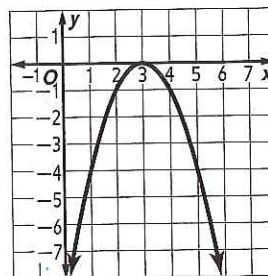
**TEKS** A.2(A) **MP** A.1(D)

$\{(1, 2), (3, 4), (5, 6), (7, 8)\}$

- A all real numbers  
 B all even numbers  
 C  $\{2, 4, 6, 8\}$   
 D  $\{1, 3, 5, 7\}$

4. What is the range of the function graphed below?

**TEKS** A.2(A) **MP** A.1(D)



- F all real numbers  
 G  $y \leq 0$   
 H all positive real numbers  
 J  $x > 0$

5. What is the range of  $y = -x + 3$  if the domain is  $\{-2, 0, 2, 4\}$ ? **TEKS** A.2(A) **MP** A.1(D)

- A  $\{3\}$   
 B  $\{-1, 1, 3, 5\}$   
 C  $\{1, 3, 5\}$   
 D  $\{1, 3, 5, 7\}$

# Preparing for Assessment

## Cumulative Assessment



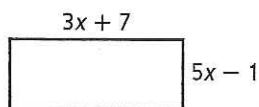
Go Online! for  
Standardized  
Test Practice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A box of fruit has 12 plums and 15 bananas. The cost of each plum is  $p$  and the cost of each banana is 25 cents. What equation gives the total cost  $C$  for all of the fruit in the box? **TEKS** A2(C)

**MP** A.1(A), A.1(E)

- A  $C = 12 \cdot 0.25 + 15p$   
 B  $C = 12 + p + 0.25 + b$   
 C  $C = 12p + 15 \cdot 0.25$   
 D  $C = 12 \cdot 15 + p \cdot 0.25$
2. A park has the dimensions indicated in the diagram.



Which equation represents the perimeter  $P$  of the park? **TEKS** A.5(A) **MP** A.1(A), A.1(E)

- F  $P = 8x + 6$   
 G  $P = 16x + 12$   
 H  $P = 6(10x + 2)$   
 J  $P = 15x^2 - 7$
3. Which of the following does not represent a function? **TEKS** A.12(A) **MP** A.1(G)

A  $y = x + 2$

B  $\{(3, 5), (5, 8), (8, 10), (3, 12)\}$

C

$x$	1	2	3	4
$y$	-4	0	-4	0

D  $f(x) = 4 - 3x^2$

4. A lunch counter serves soup in 12-ounce containers. Which of the following represents an equation and a solution for the total ounces of soup if 150 containers are sold? **TEKS** A.5(A) **MP** A.1(A), A.1(E)

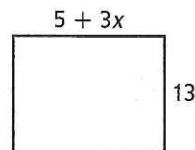
F  $150s = 12; s = 12.5$

G  $12 + s = 150; s = 138$

H  $\frac{s}{12} = 150; s = 180$

J  $s = 150 \times 12; s = 1800$

5. The diagram shows the dimensions of a rectangle. Each side is measured in centimeters.



Which equation represents the area  $A$  of the rectangle? **TEKS** A.10(D) **MP** A.1(A)

A  $A = 10 + 32x \text{ cm}^2$

B  $A = 18 + 3x \text{ cm}^2$

C  $A = 36 + 6x \text{ cm}^2$

D  $A = 65 + 39x \text{ cm}^2$

6. The table shows a relation.

$x$	-3	5	5	6	7
$y$	4	-2	3	6	-2

Which set of ordered pairs represents the same relation? **TEKS** A.2(A) **MP** A.1(D), A.1(E)

F  $\{-3, -2, 3, 4, 5, 6, 7\}$

G  $\{(-3, 4), (6, 6), (7, -2)\}$

H  $\{(4, -3), (-2, 5), (3, 5), (6, 6), (-2, 7)\}$

J  $\{(-3, 4), (5, -2), (5, 3), (6, 6), (7, -2)\}$

7. What is the solution of  $m = 3 - \frac{1}{2}m$ ? **TEKS** A.5(A) **MP** A.1(D)

A -2

B  $\frac{3}{4}$

C 2

D 6

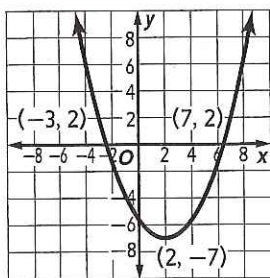
### Test-Taking Tip

**Question 6** The  $x$ -values represent the domain values, or the first coordinates, of a relation. In a table, the top or left row generally has the first coordinates of the ordered pairs.

8. For the function  $y = 15 - 6x$ , suppose the domain is only values of  $x$  from 10 to 20. What is the range of the function? **TEKS** A.2(A) **MP** A.1(F)

- F All values from 45 to 105  
 G All values from  $-\frac{5}{6}$  to  $\frac{5}{6}$   
 H All values from  $-105$  to  $-45$   
 J The two values  $-105$  and  $-45$

9. Which statement best describes the symmetry of the graph? **TEKS** A.7(A) **MP** A.1(E)



- A The graph is a mirror image of itself over the  $x$ -axis.  
 B The graph is a mirror image of itself over the line  $y = -7$ .  
 C The graph is a mirror image of itself over the line  $x = 2$ .  
 D The graph does not exhibit any symmetry.
10. At a coffee shop, a small cup contains 8 fluid ounces and a large cup contains 14 fluid ounces. How many fluid ounces are there in 15 small cups and 20 large cups? **TEKS** A.5(A) **MP** A.1(A), A.1(D)
- F 370 fluid ounces  
 G 400 fluid ounces  
 H 412 fluid ounces  
 J 782 fluid ounces

11. For the statement  $3.2n = 19.2$ , what is the value of  $n$  and what property is used to find  $n$ ? **TEKS** A.5(A) **MP** A.1(G)

- A  $n = 6$ , Additive Inverse Property  
 B  $n = 6$ , Multiplicative Inverse Property  
 C  $n = 6$ , Substitution  
 D  $n = 16$ , Multiplicative Inverse Property

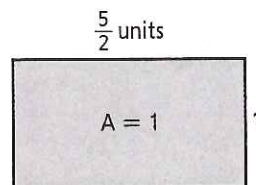
12. What is a simplified form of the expression  $(15k + 2) - (k^2 - 2)$ ? **TEKS** A.10(D) **MP** A.1(D)

- F  $14k + 4$                       H  $-k^2 + 15k + 4$   
 G  $-k^2 + 15k$                     J  $k^2 + 15k + 4$

13. Use the order of operations to simplify  $7 \cdot 3 - 5x + 2 = 5^2 + 2x - 7x - 2$ . What is the solution? **TEKS** A.5(A) **MP** A.1(G)

- A  $23 - 5x = 23 - 5x$ ; no solution  
 B  $23 - 5x = 23 - 5x$ ; all real numbers  
 C  $23 - 5x = 25 - 5x$ ; no solution  
 D  $23 - 5x = 25 - 5x$ ; all real numbers

14. **GRIDDABLE** The area of this rectangle is 1 square unit. Using the Multiplicative Inverse Property, what is the unknown dimension of the rectangle, expressed as a decimal? **TEKS** A.5(A) **MP** A.1(G)



15. The ordered pairs  $\{(-2, 2), (1, 2), (3, -1), (5, 2), (5, 4)\}$  represent a relation that is not a function. Which change would make it a function? **TEKS** A.12(A) **MP** A.1(B), A.1(D)

- F Replace  $(5, 2)$  with  $(3, 2)$ .  
 G Replace  $(5, 4)$  with  $(1, 5)$ .  
 H Replace  $(5, 2)$  with  $(5, -3)$ .  
 J Replace  $(5, 4)$  with  $(4, 5)$ .

### Need Extra Help?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	1-1	1-2	1-7	1-5	1-4	1-6	1-1	1-7	1-8	1-2	1-3	1-4	1-5	1-3	1-7