1-5 Equations

Find the solution set of each equation if the replacement sets are *y*: {1, 3, 5, 7, 9} and *z*: {10, 12, 14, 16, 18}.

16. $17 = 24 - y$

SOLUTION:

y	$17 = 24 - y$	True or False?
1	$17 = 24 - 1$	False
3	$17 = 24 - 3$	False
5	$17 = 24 - 5$	False
7	$17 = 24 - 7$	True
9	$17 = 24 - 9$	False

The solution set is {7}.

17. $2z - 5 = 27$

SOLUTION:

z	$2z - 5 = 27$	True or False?
10	$2(10) - 5 = 27$	False
12	$2(12) - 5 = 27$	False
14	$2(14) - 5 = 27$	False
16	$2(16) - 5 = 27$	True
18	$2(18) - 5 = 27$	False

The solution set is {16}.

Solve each equation.

26. $a = \frac{4(14-1)}{3(6)-5} + 7$

SOLUTION:

$$a = \frac{4(14-1)}{3(6)-5} + 7 \quad \text{Original equation}$$

$$a = \frac{4(13)}{18-5} + 7 \quad \text{Subtract 1 from 14.}$$

$$a = \frac{52}{18-5} + 7 \quad \text{Multiply 4 by 13.}$$

$$a = \frac{52}{13} + 7 \quad \text{Subtract 5 from 18.}$$

$$a = 4 + 7 \quad \text{Divide 52 by 13.}$$

$$a = 11 \quad \text{Add 4 and 7.}$$

28. $7+x-(3+32\div 8)=3$

SOLUTION:

$$\begin{aligned}
7+x-(3+32\div 8)&=3 && \text{Original equation}\\
7+x-(3+4)&=3 && \text{Divide 32 by 8.}\\
7+x-7&=3 && \text{Add 3 and 4.}\\
7+(-7)+x&=3 && \text{Commutative Property.}\\
0+x&=3 && \text{Subtract 7 from 7.}\\
x&=3 && \text{Simplify.}
\end{aligned}$$

30. $(3\cdot 6\div 2)v+10=3^2v+9$

SOLUTION:

$$\begin{aligned}
(3\cdot 6\div 2)v+10&=3^2v+9 && \text{Original equation}\\
(18\div 2)v+10&=3^2v+9 && \text{Multiply 3 by 6.}\\
(18\div 2)v+10&=9v+9 && \text{Evaluate power.}\\
9v+10&=9v+9 && \text{Divide 18 by 2.}
\end{aligned}$$

No matter what real value is substituted for v, the left side of the equation will always be one more than the right side of the equation. So, the equation will never be true, and there is no solution.

32. $(3\cdot 5)t+(21-12)=15t+3^2$

SOLUTION:

$$\begin{aligned}
(3\cdot 5)t+(21-12)&=15t+3^2 && \text{Original equation}\\
15t+9&=15t+9 && \text{Evaluate power.}\\
15t+9&=15t+9 && \text{Multiply 2 by 3.}\\
15t+9&=15t+9 && \text{Subtract 24 from 27.}
\end{aligned}$$

No matter what value is substituted for h, the left side of the equation will always be equal to the right side of the equation. So, the equation will always be true. The solution is all real numbers.

1-5 Equations

34. $\frac{3 \cdot 22}{18+4}r - \left(\frac{4^2}{9+7} - 1\right) = r + \left(\frac{8 \cdot 9}{3} \div 3\right)$

SOLUTION:

$$\frac{3 \cdot 22}{18+4}r - \left(\frac{4^2}{9+7} - 1\right) = r + \left(\frac{8 \cdot 9}{3} \div 3\right)$$

$$\frac{66}{18+4}r - \left(\frac{4^2}{9+7} - 1\right) = r + \left(\frac{8 \cdot 9}{3} \div 3\right) \quad \text{Multiply.}$$

$$\frac{66}{18+4}r - \left(\frac{16}{9+7} - 1\right) = r + \left(\frac{8 \cdot 9}{3} \div 3\right) \quad 4^2 = 16$$

$$\frac{66}{18+4}r - \left(\frac{16}{16} - 1\right) = r + \left(\frac{8 \cdot 9}{3} \div 3\right) \quad \text{Add.}$$

$$\frac{66}{18+4}r - \left(\frac{16}{16} - 1\right) = r + \left(\frac{72}{3} \div 3\right) \quad \text{Multiply.}$$

$$\frac{66}{22}r - \left(\frac{16}{16} - 1\right) = r + \left(\frac{72}{3} \div 3\right) \quad \text{Add.}$$

$$3r - (1-1) = r + \left(\frac{72}{3} \div 3\right) \quad \text{Divide.}$$

$$3r - (1-1) = r + (24 \div 3) \quad \text{Divide.}$$

$$3r - 0 = r + (24 \div 3) \quad \text{Subtract.}$$

$$3r - 0 = r + 8 \quad \text{Divide.}$$

$$3r = r + 8 \quad \text{Simplify.}$$

Test values of *r* for which the statement is true.

$$\begin{array}{ccc} 3(0) & \stackrel{?}{=} & 0+8 \\ 0 & \neq & 8 \\ 3(2) & \stackrel{?}{=} & 2+8 \\ 6 & \neq & 10 \\ 3(4) & \stackrel{?}{=} & 4+8 \\ 12 & = & 12 \end{array}$$

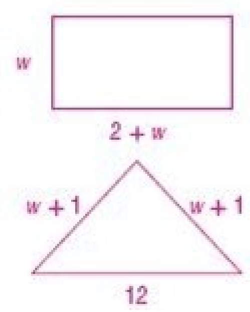
The only value for *r* that makes the equation true is 4. So, *r* = 4.

63. **GEOMETRY** The length of a rectangle is 2 inches greater than the width. The length of the base of an isosceles triangle is 12 inches, and the lengths of the other two sides are 1 inch greater than the width of the rectangle.

a. Draw a picture of each figure and label the dimensions.
b. Write two expressions to find the perimeters of the rectangle and triangle.
c. Find the width of the rectangle if the perimeters of the figures are equal.

SOLUTION:

a.



b. The formula for the perimeter of a rectangle is $P = 2l + 2w$.

$$\begin{aligned} P &= 2(2+w)+2w \\ &= 2(2)+2w+2w \\ &= 4+(2+2)w \\ &= 4+4w \end{aligned}$$

The formula for the perimeter of a triangle is $P = a + b + c$.

$$\begin{aligned} P &= (w+1)+(w+1)+12 \\ &= w+w+1+1+12 \\ &= (1+1)w+14 \\ &= 2w+14 \end{aligned}$$

c. Because the perimeters are equal, set the expression from parts **a** and **b** equal to each other and solve for w.

$$\begin{aligned} 4w+4 &= 2(w+1)+12 \\ 4w+4 &= 2w+2+12 \\ 4w+4 &= 2w+14 \end{aligned}$$

Test values for w.

$$4(1)+4 \stackrel{?}{=} 2(1)+14$$
$$8 \neq 16$$
$$4(2)+4 \stackrel{?}{=} 2(2)+14$$
$$12 \neq 18$$
$$4(4)+4 \stackrel{?}{=} 2(4)+14$$
$$20 \neq 22$$
$$4(5)+4 \stackrel{?}{=} 2(5)+14$$
$$24 = 24$$

The only value of w that makes the equation true is 5. So, $w = 5$ inches.

69. **ERROR ANALYSIS** Tom and Li-Cheng are solving the equation $x = 4(3 - 2) + 6 \div 8$. Is either of them correct? Explain your reasoning.

Tom

$$
\begin{aligned}
x &= 4(3-2) + 6 \div 8 \\
&= 4(1) + 6 \div 8 \\
&= 4 + 6 \div 8 \\
&= 4 + \frac{6}{8} \\
&= 4\frac{3}{4}
\end{aligned}
$$

Li-Cheng

$$
\begin{aligned}
x &= 4(3-2) + 6 \div 8 \\
&= 4(1) + 6 \div 8 \\
&= 4 + 6 \div 8 \\
&= 10 \div 8 \\
&= \frac{5}{4}
\end{aligned}
$$

SOLUTION:

Tom; Tom evaluated inside the parenthesis first. Then he preformed multiplication and then division. Finally Tom added. Li-Cheng did evaluate inside the parenthesis first. However, next, she added 6 + 4 instead of dividing 6 by 8. She did not follow the order of operations.

70. **PROBLEM SOLVING** Find all of the solutions of $x^2 + 5 = 30$

SOLUTION:

For the equation $x^2 + 5 = 30$ to be true, the value of x^2 must be 25. Both 5^2 and $(-5)^2$ result in 25. So, the equation has two solutions, 5 and −5.

1-5 Equations

73. What is the solution to the equation $t = 5 - (3^2 + 1) + 2$?

A –3
B –1
C 0
D 17

SOLUTION:

$$\begin{aligned} t &= 5 - (3^2 + 1) + 2 \\ &= 5 - (9 + 1) + 2 \\ &= 5 - 10 + 2 \\ &= -5 + 2 \\ &= -3 \end{aligned}$$

The correct answer is choice A.

77. The total resistance T in ohms of two resistors with resistance R_1 and R_2 is given by the equation:

$$\frac{1}{T} = \frac{1}{R_1} + \frac{1}{R_2}$$

If the total resistance is 12 ohms and one resistor has a resistance of 30 ohms, what is the resistance of the other resistor?

A $\frac{1}{12}$ ohm
B 2.5 ohms
C 20 ohms
D 42 ohms

SOLUTION:

$$\begin{aligned} \frac{1}{T} &= \frac{1}{R_1} + \frac{1}{R_2} \\ \frac{1}{12} &= \frac{1}{30} + \frac{1}{R_2} \\ \frac{5}{60} &= \frac{2}{60} + \frac{1}{R_2} \\ \frac{3}{60} &= \frac{1}{R_2} \\ \frac{1}{20} &= \frac{1}{R_2} \\ 20 &= R_2 \end{aligned}$$

The resistance of the other resistor is 20 ohms, so choice C is the correct answer.